Unconventional Monetary Policy and the Bond Market in Japan: A New-Keynesian Perspective

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Abstract

In this paper, we set up a medium scale new-Keynesian dynamic stochastic general equilibrium (DSGE) model to analyze the effects of various phases of unconventional monetary policy (UMP) on the Japanese bond market. Our model has two novel features: (i) a banking friction in the form of an aggregate bank run risk to motivate a commercial banks' demand for excess reserve, and (ii) dynamic linkage between Central Bank resource constraint and the government budget constraint via a transfer payment by the Central Bank to the Treasury. We do three policy simulations to analyze the effects of various phases of UMP shocks on the bond market, namely: (i) effect of a quantitative easing (QE) shock; (ii) the effect of negative shock to the overnight borrowing rate; and (iii) the effect of a negative shock to the interest rate on banks' excess reserve (IOER). In light of these results, we do a policy evaluation of the recent yield curve control policy of the Bank of Japan.

Keywords: QE; QQE; Excess Reserve; Overnight Borrowing Rate; IOER; Yield Curve Control

JEL classification: E43, E44, E58

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1 Introduction

During the last two decades, the Japanese economy experienced several episodes of monetary policy changes. Starting from an era of near zero interest rate, from the beginning of the millennium, Bank of Japan officially implemented Quantitative Easing (QE) policy to inject liquidity into the banking system. After the Great Recession, with a growing concern of a deflationary bubble, Bank of Japan adopted a monetary policy with an explicit two percent inflation target which is the cornerstone of Qualitative and Quantitative Easing (QQE). QQE is a broader unconventional monetary policy (UMP) which combines three features in a chronological order: (i) quantitative easing with a two percent inflation target, (ii) negative interest rate on bank reserve and (iii) zero nominal long term bond yield target.

The aim of this research is to analyze the effects of these various episodes of UMP on the bond market behaviour in a dynamic stochastic general equilibrium (DSGE) framework. In the context of Japan, understanding the bond market implications is important because domestic bonds constitute about 30.48% of the Government Pension Investment Funds in Japan (June 2017; www.gpif.go.jp). In addition, about 85% of the total assets of Bank of Japan (BoJ) is in government bonds (August 2017; www.boj.or.jp). Also, the amount of government debt outstanding at the end of March 2017 was about 845 trillion yen. If the price of government bond falls and the yield rises, the Japanese government might not be able to refinance the current debt. Fluctuations of bond prices and yields have major implications for commercial banks and the BoJ balance sheet given that about 39.5% of government bond is held by BoJ and 20.9% is held by commercial banks and securities houses at the end of March 2017.

Our model is a stylized medium scale new Keynesian model similar in spirit to extant models such as Smets and Wouters (2007), and Gerali et al. (2010). The advantage of using DSGE model is that it enables us to see the linkage between the real and financial sectors of the economy when a policy change occurs. We derive the pricing kernel which we use to price general fixed income securities, its yield to maturity and term premia. Our aim is to analyze the impulse responses of yield to maturity, holding period returns and term premia of long term government bonds to various monetary shocks pertaining to QQE.

As in any standard DSGE model, our model economy has the following decision units: (i) representative household, (ii) firms, (iii) capital good producers, (iv) wholesale good producers, (v) retail good producers, (vi) banking sector and the (vii) Central Bank (CB hereafter) and the government. The model has standard frictions such as aggregate habit persistence, investment adjustment cost, monopolistic price formation and nominal stickiness. A non-standard feature of our DSGE model is the banking friction in the form of an aggregate bank run risk as in Chang et al. (2014). Banks anticipate the risk of an excess withdrawal of deposits over its reserve which could force them to take recourse to emergency overnight borrowing from the CB at a penal rate. Such an anticipation disciplines the banks to hold precautionary excess reserve and not push loans recklessly. This precautionary demand for bank reserve depends on the overnight borrowing rate and the interest rate on reserve making these two rates CB policy tools.

A novel feature of our study is that we explicitly study the link between the government, CB and commercial banks via the long term government bonds and bank reserves which helps us study the effect of QQE policy on the bond market. We formulate the CB's resource constraint in line with the recent work of Hall and Reis (2015) which characterizes the supply function of bank reserve. CB creates reserve (which is the monetary base) keeping in mind that it has to pay interest and principal on existing bank reserves, and also cover the net QE purchase of government bonds from commercial banks. In addition, CB pays some transfer to the government after netting out the revenue that it receives from banks as a penalty from overnight loans. This CB transfer to the government helps finance the current budget deficit of the Treasury subject to the outstanding debt/GDP ratio. This dynamic linkage between the CB resource constraint, commercial bank's flow budget constraint and the government budget constraint is crucial for understanding the monetary transmission mechanism of UMP.

In the extant literature, the real effects of QE arise from limit to arbitrage. This limit to arbitrage can arise from lender's moral hazard as in Gerter and Karadi (2013) or some market segmentation due to preferred habitat (Vyanos and Villa, 2009) or transaction cost as in Chen et al. (2012). We model market segmentation in a simple way within the framework of frictionless financial markets. Households in our model do not trade in long term government bonds and hold short term bank deposits which are perfect substitutes of short term government bonds. Bank deposits provide direct convenience utility to households as in Hansen and Imrohoroglu (2013) in addition to transaction utility of money. This assumption gives rise to a natural steady state borrowing-lending spread in our model. On the other hand, commercial banks specialize in dealing with long term government bonds and loans. The assumption of bond market segmentation in our model is motivated by the Japanese financial structure where commercial banks and the Central Bank are primary dealers of government bonds issued by the Treasury.

We formulate the basic QQE operation as a stochastic open market operation with a two percent long run inflation target and a target debt/GDP ratio. Such a policy entails a positive shock to monetary base with an offsetting increase in the share of CB holding of long term Japanese government bonds (JGB). Higher inflationary expectations triggered by a positive monetary base shock entails an inflation tax on banking excess reserve. Commercial banks thus lower excess reserve and issue more loans. In equilibrium, the nominal loan rate also rises together with higher investment in response to inflationary expectations. On the other hand, the price of long term bond falls raising the nominal yield to maturity. The overall effect of QE is thus positive on the economy. We compare UMP with the conventional monetary policy (CMP) where CMP is modelled as a standard Taylor rule shock to the overnight borrowing rate in our model. Such a policy rate directly affects the precautionary demand for reserve by the commercial banks and through this channel it impacts the bond yields via the stochastic discount factor. We find that the real effect of a QQE shock is considerably stronger than a CMP shock. However, a QQE shock is unable to predict the observed decline in the nominal yield to maturity while a CMP predicts such a decline but rather weakly in terms of magnitude.

We also study the effects of UMP and CMP on the term premia. Recently a few papers examine the effects of large scale asset purchases (LSAP) of US Federal Reserve on term premia. Gagnon et al. (2011) use a reduced form term premium equation to assess the effect of LSAP on term premia. Chen et al. (2012) use a DSGE model to analyze the same effect but they do not model the Central Bank balance sheet and link it to the Central Bank purchase of government bond as we do. As in Gagnon et al. (2011) and Chen et al. (2012), we find that the Central Bank's bond purchase programme lowers the term premia of bonds of all maturities but shorter maturity bonds experience a sharper decline in term premium. We further investigate the effect of a negative shock to the interest rate on excess reserve (IOER) to depict the recent QQE experiment in Japan. We find that a negative IOER shock lowers the term premia and its effect is more pronounced than a positive QE shock. In addition, such a drop in IOER stimulates the aggregate economy through bank lending channel because banks loan out their excess reserve to avoid penalty which stimulates investment. On the other hand, an expansionary CMP shock has near zero effect on term premia.

Our DSGE model predicts that a negative IOER shock is the most effective way to stimulate the economy within the framework of the recent yield curve control experiment of BoJ with a positive inflation target. Nominal yield to maturity declines while inflation rises marginally in response to such negative IOER shock. However, such a policy might work only in the short run. We find that targeting a zero long term yield is inconsistent with a two percent long run inflation target policy because it violates the fundamental Fisher's relationship.

The paper is organized as follows. In the next section, we briefly review the extant literature on DSGE modelling of the Japanese economy. In section 3, we give an overview of the monetary policy history of Japan. In section 4, we present some relevant business cycle stylized facts. In section 5, our basic DSGE model is laid out. Section 6 is devoted to present quantitative analysis of the model. Section 7 concludes.

2 Connections to Literature

There is a growing literature on DSGE modelling of the Japanese economy. The literature can be broadly classified in two strands: (i) with no explicit zero lower bound on the nominal interest rate and (ii) with zero lower bound on the interest rate. In the first group, Sugo and Ueda (2007) is one of the first articles that estimate a DSGE model of the Japanese economy. Although they model monetary policy rule and use call rate as a proxy for the short term nominal interest rate, they do not explicitly model the role of CB and abstract from any analysis of monetary or fiscal policy effects on bond market except that there is an interest rate shock through a discount bond. Iwata (2009) focuses on the fiscal policy under DSGE setting. Hirose (2014) estimates a DSGE with a deflationary steady state for Japan and considers whether several shocks to the economy have an inflationary effect. McNelis and Yoshino (2016) compare the performance of three policy rules on reducing the government debt using a DSGE model. However, they do not explicitly model the role of CB and there is no government bond in the model. Fueki et al. (2016) set up a DSGE model to analyze potential output and output gap for the Japanese economy.

In the second group of literature, Adjemian and Julliard (2010) estimate a DSGE model with zero lower bound for nominal interest rate. Michaelis and Watzka (2017) consider the change in the effectiveness of quantitative easing policy at the zero lower bound. Although there are liquidity shocks in their model, they do not have a DSGE model. Instead they estimate time varying parameter using VAR analysis and do not study the effect of monetary policy on bonds.

Our study contributes more to the first strand of literature. In contrast with extant studies, we explicitly model the role of CB and the nexus between the government budget constraint, the CB budget constraint and the commercial bank's flow budget constraint. We focus on the transmission channels of QQE policy of BoJ to the Japanese bond market via the dynamic linkage between CB and government budget constraints which is a new contribution in the literature.

While a plethora of literature exists on various applications of DSGE models, what is less understood is its bond yield implications. Rudebusch and Swanson (2012) show some innovative applications of DSGE model to understand bond pricing behaviour. However, they do not focus on the monetary policy effects on the bond market behaviour, which is the scope of our study. Chen et al. (2012) is one of the few studies that uses DSGE modelling to assess the effects of UMP on long term bond yields in the US who find that the QE in the US has rather insignificant effects on long term bond yield. They, however, do not formulate the CB balance sheet and commercial banks' asset portfolio which we do. Moreover, their focus is on the QE operation in the US, while our focus is on QQE in Japan which involves additional monetary policy instruments including IOER. As in Chen et al.(2012), our model also predicts that a simple QE in the form of CB's open market purchase of long term government bonds has insignificant effect on the long term bond yield. However, we find that QE has a nontrivial effects on the term premia. In addition, we find that a negative shock to the interest rate on excess reserve has a stronger effect on the term premium as opposed to QE. We also find that a CMP in the form of an overnight borrowing rate shock has near zero effect on bond yield and term premia.

3 An Overview of Bank of Japan Monetary Policy

3.1 Background

Bank of Japan (BoJ) changed the policy rate from the official discount rate to short term market rate in March 1995, although they were not explicit about the exact short term rate as the operating target. From 1998, BoJ specified the uncollateralized overnight call rate as the policy rate and they also started to announce the target level for the call rate. On February 12, 1999, the BoJ decided to take the following two actions: (i) Provide ample liquidity and encourage the uncollateralized overnight call rate to move as low as possible, and (ii) Guide the call rate to move around 0.15%, and subsequently induce further decline in view of the market developments.[1] The well known "zero interest rate" policy was announced, although the initial target call rate was at 0.15%. The call rate dropped from around 0.3% before the announcement to around 0.05% after the announcement. On August 11, 2000, BoJ decided to lift the zero interest policy by stating that the Bank of Japan will encourage the uncollateralized overnight call rate to float on the average around 0.25%.[2] The call rate surged from around 0.05% before the announcement to around 0.3% after the announcement. Figure 1 shows a plot of the overnight call rate.[3]

[1] This is based on the "Announcement of the Monetary Policy Meeting Decisions" released on February 12, 1999.

[2] This is based on the press release on August 11, 2000, titled "Change of the Guideline for Money Market Operations."

[3] The appendix provides the details of all data sources.

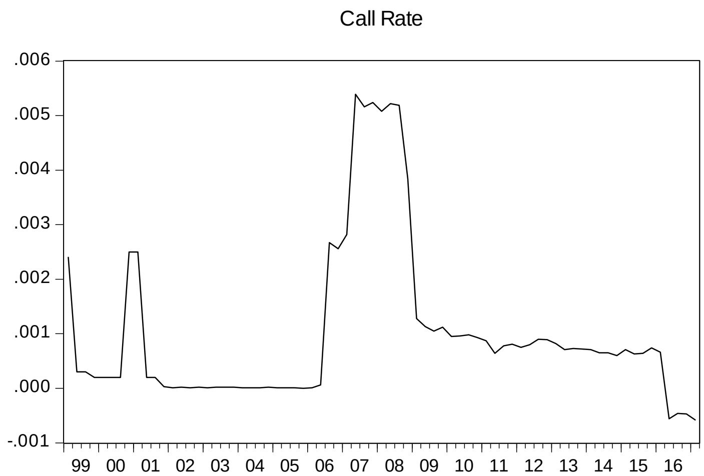

Figure 1: Overnight Call Rate

3.2 Quantitative Monetary Easing Policy (QME)

On March 19, 2001 the Bank of Japan announced four major changes in monetary policy :[4] (i) The main operating target for money market operations was changed from the current uncollateralized overnight call rate to the outstanding balance of the current accounts at BoJ;[5] (ii) The new procedures for money market operations would continue until a zero percent CPI annual inflation target is achieved, (iii) The balance outstanding at BoJ's current accounts would increase by 1 trillion yen from the average outstanding of 4 trillion yen in February 2001, and (iv) The BoJ would increase the amount of its outright purchase of long-term government bonds from the current 400 billion yen per month to provide liquidity smoothly to the private sector.

With this announcement, the call rate spiked and then the level of call rate dropped to almost zero as seen in Figure 1. The yield for long term 10 year JGB showed a mixed reaction to this announcement. It kept rising for more than four weeks after the announcement. After that the yield dropped gradually with occasional bumps but it rose sharply later after mid 2015 as seen in Figure 2.

[4] See the press release published on March 19, 2001, titled "New Procedures for Money Market Operations and Monetary Easing."

[5] We use the phrase current and reserve synonymously here. In practice there is a difference of the order of 2%.

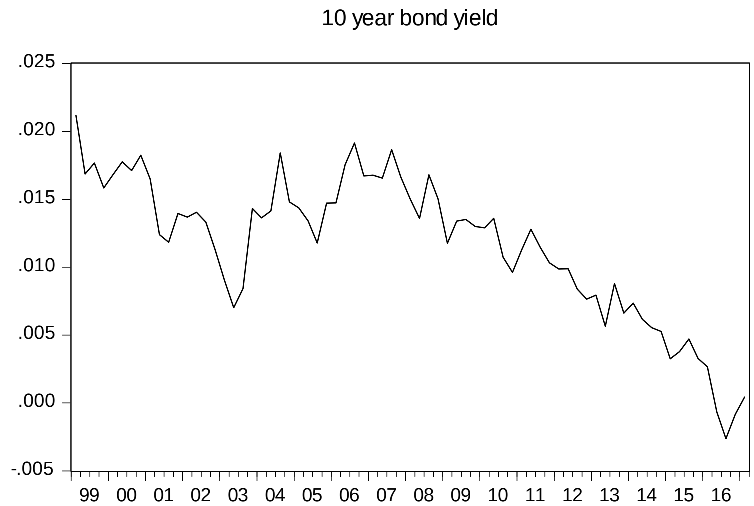

Figure 2: 10 Year JGB Bond Yield

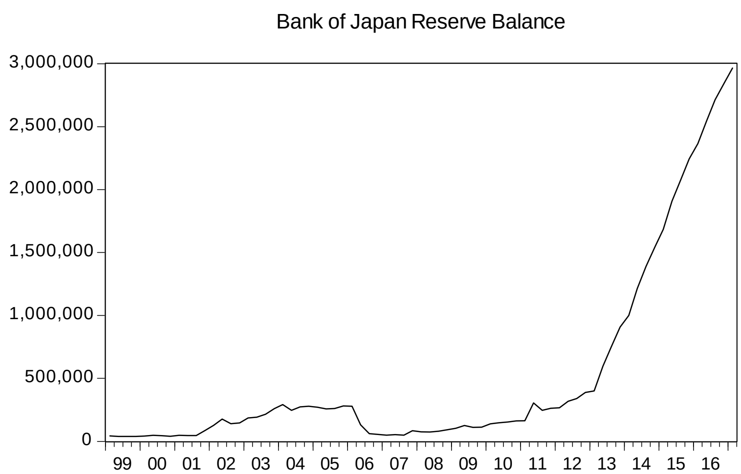

Figure 3: Bank Reserve

On March 9, 2006 BoJ changed the operating target from the reserve to uncollateralized overnight call rate.[6] After the announcement, there was a gradual increase in the level of the overnight call rate. The reserve balance also declined initially reaching 7.6 trillion yen in May 2008 but it has increased afterwards reaching 58.1 trillion yen in March 2013. Figure 3 plots the time series of BoJ reserve balance.

[6] Based on a press release published on March 9, 2006, titled "Change in the Guideline for Money Market Operations."

3.3 Qualitative and Quantitative Monetary Easing Policy

On April 4, 2013, the BoJ decided to introduce Qualitative and Quantitative Monetary Easing Policy (QQE). The main objective was to achieve an annual CPI inflation target of 2 percent. In order to achieve this objective BoJ decided to implement the "monetary base control" [7] with an announcement to double the monetary base and the amounts outstanding of Japanese government bonds (JGB) in two years, and more than double the average remaining maturity of JGB purchases. The amount of monetary base nearly doubled between April 2013 and April 2015 (Figure 4). Similar trend was reflected in the reserve balances held at the BoJ by banks as seen in Figure 3. From 2009 onward, long term bonds held by BoJ exponentially increased while the holding of the same by the banks decreased. This reflects a large scale purchase of long term government bonds by the BoJ which is the very essence of QQE (Figure 5).

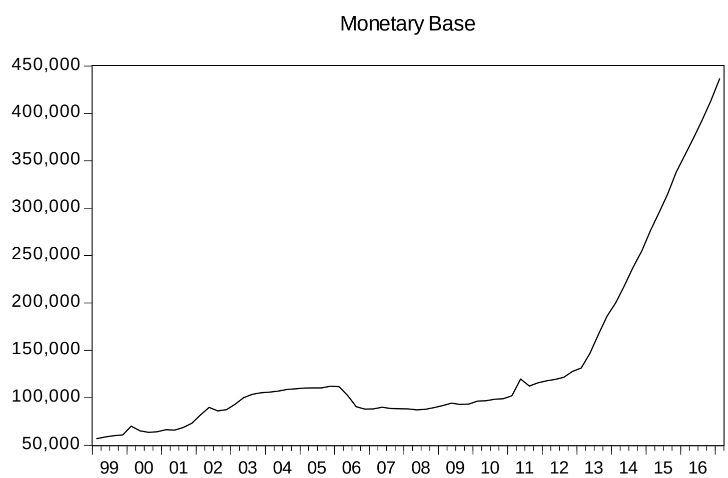

Figure 4: Monetary Base

[7] See the press release published on April 4, 2013, "Introduction of the Quantitative and Qualitative Monetary Easing."

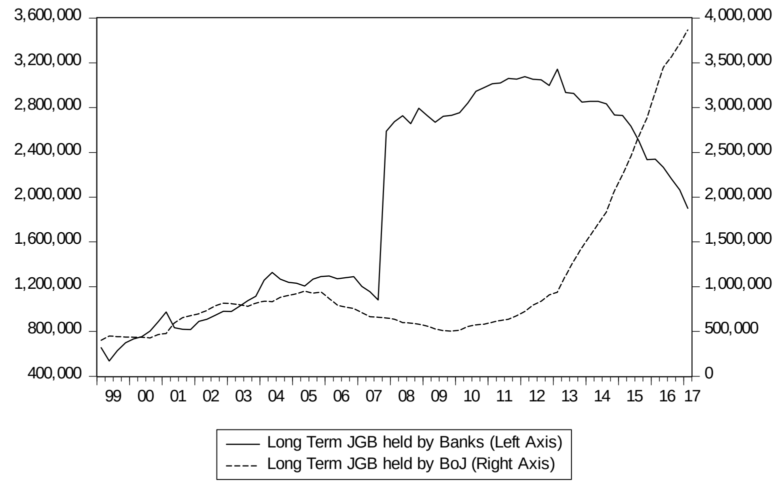

Figure 5: Long Term JGB Holding

3.4 Qualitative and Quantitative Monetary Easing Policy with Negative Interest Rate

On January 29, 2016, BoJ decided to implement Quantitative and Qualitative Monetary Easing (QQE) with a negative Interest rate on excess reserve. In order to achieve the price stability target of two percent at the earliest possible time, the BoJ set the negative interest rate on current accounts at -0.1%.[8] They also adopted a three-tier system in which the outstanding balance of each bank's current account at the Bank will be divided into three tiers offering positive, zero and negative interest rates respectively.[9]

3.5 Qualitative and Quantitative Monetary Easing Policy with Yield Curve Control

On September 21, 2016, the BoJ introduced a new policy framework that consists of two major components: the first is "yield curve control" in which the Bank will control short-term and long-term interest rates; and the second is an "inflation-overshooting commitment" in which the Bank commits itself to expanding the monetary base until the year-to-year CPI inflation rate exceeds the inflation target of two percent and stays above the target in a stable manner.[10]

[8] See the press release published on January 29, 2016, titled 'Introduction of "Quantitative and Qualitative Monetary Easing with a Negative Interest Rate".'

[9] See the same report publised on January 29, 2016.

[10] See the press release published on September 21, 2016, titled "New Framework for Strengthening Monetary Easing: Quantitative and Qualitative Monetary Easing with Yield Curve Control."

4 Business Cycle Stylized Facts

Figures 6 plots the business cycle components of the basic macroeconomic aggregates, namely GDP, consumption, investment and employment while Figure 7 plots the CPI inflation series based on the raw data. All these macroeconomic variables are per capita seasonally adjusted series deflated by CPI (all items). To identify the business cycle components, we pass each level series through an asymmetric band pass filter prescribed by Christiano and Fitzerald (2003). All four series generally move together. What is noteworthy is that there is no clear correspondence between business cycle upswings and quantitative easing as one may expect. GDP initially jumped when the QE was first launched in 2001. Since then GDP fluctuated a great deal around the trend. GDP series went appreciably above the trend line between 2005 and 2008 when BoJ was phasing out QE and was in the process of switching to interest rate target. After recovering from the Great Recession, from 2010 onward, the economy was generally on an upward trajectory. Since 2015, GDP was above the trend which is curiously the QQE period with two percent inflation and negative interest rate targets. What is also noteworthy is that consumption was on a rising trajectory with occasional bumps above the trend line while there was a deceleration of investment till 2009 after which it picked up.

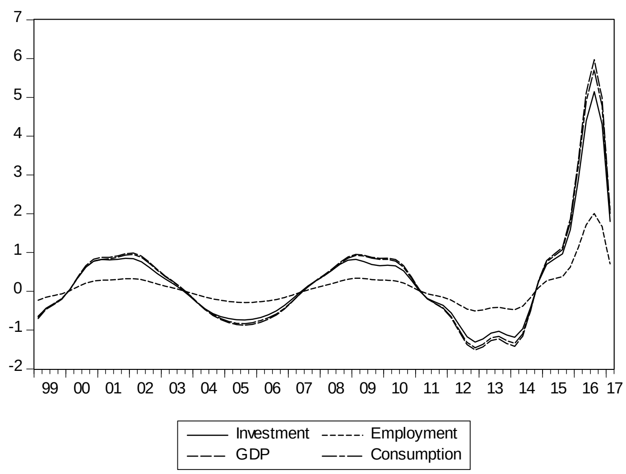

Figure 6: Business Cycle Components of Macroeconomic Real Aggregates

Figure 7 plots the CPI inflation series. What is noteworthy is the revival of inflation from zero percent since 2012. In 2014, it exceeded the QQE two percent target first time in the history. Since then it again decelerated and converged to zero percent rate.[11]

[11] This inflation hike may be overstated because in Japan consumption tax was introduced in 1989. The initial tax rate was 3%. It was raised to 5% on April 1 1997 and to 8% on April 1 2014. If one uses the tax

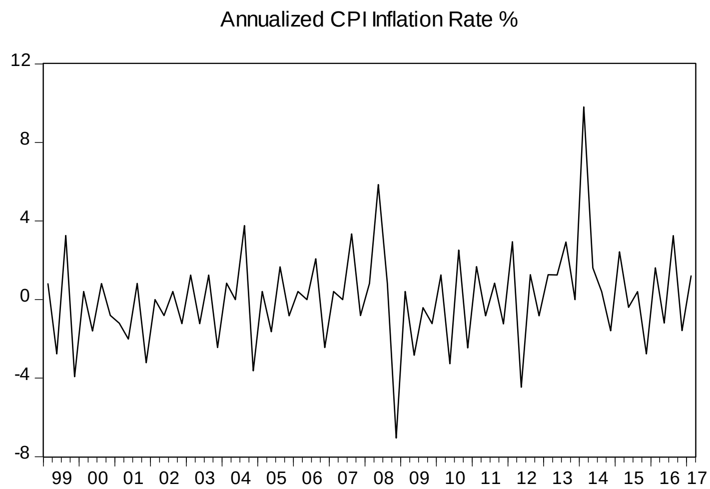

Figure 7: CPI Inflation (%)

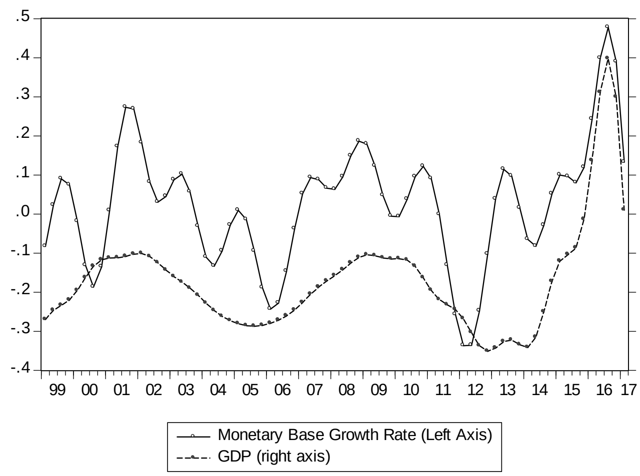

Figure 8: Business Cycle Components of Monetary Base Growth Rate and GDP

adjusted CPI index released on May 26, 2017 by Economic and Social Research Institute, Cabinet Office, the change in CPI is much lower.

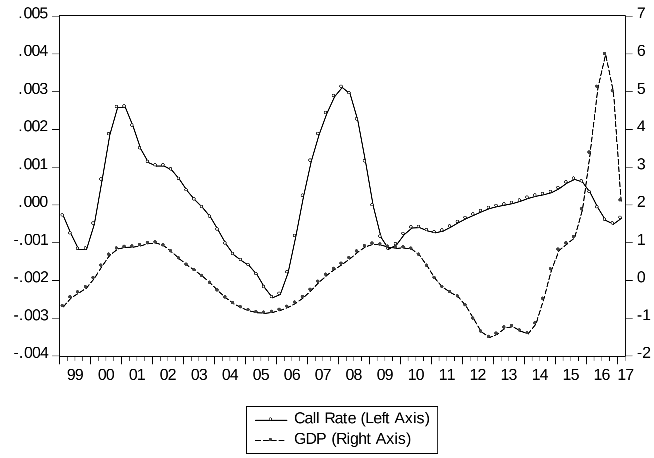

Figure 9: Business Cycle Components of Call Rate and GDP

Figure 8 plots the business cycle components of per capita real GDP and monetary base growth rate. Figure 9 plots the same GDP against the business cycle component of overnight call rate. There is a close association between monetary base growth and GDP particularly after 2001 that broadly coincides with the onset of QE era. The correlation coefficient is 0.73 which is significant at one percent level. The correlation between call rate and GDP is rather weak (0.20) and is not statistically significant at a 5% level.

The aim of our study is to primarily analyze the effect of QE and QQE on the bond market and the aggregate economy. Thus we focus our attention on the period 2001 to the current. There are basically five policy regimes during this periods: (i) March 2001-March, 2006: QE with monetary base as the operating target, (ii) March 2006-April 2013: QE phasing out period when the operating target changed from monetary base to overnight call rate, (iii) April 2013-January 2016: QQE with a 2% inflation target, (iv) January 2016-September 2016, QQE with negative interest rate, (v) September 2016-to date, QQE with yield curve control.

In the next section, we lay out a new Keynesian DSGE model to understand the effect of QE and other structural shocks on the aggregate economy and bond pricing aggregates.

5 Model

5.1 Story

We have seven players in the economy: the representative household, three types of firms, commercial banks, CB and the government . Household owns all productive units and thus profits are received as transfers to the household. Households save in the form of short term bank deposits (which are perfect substitutes for short-term government bonds) and long- term government bonds. They supply labour to wholesale goods firms. Their income consists of labour, interest income from deposit and cash flows generated from the ownership of firms and the banks.

Three types of firms are: retail, wholesale and capital goods producers. Competitive risk neutral one period lived wholesale firms finance their capital spending from banks. Competitive capital goods producers buy used capital from wholesalers and refurbish it to new capital using investment goods bought from the retail producers. Retail producers convert wholesale goods costlessly to final goods and has some monopoly power of price fixing. Final goods can be used for household consumption, capital goods producers' investment and government use.

Banks collect household deposit and intermediate this to wholesale firms and also hold long term government bonds and excess reserve since they anticipate an aggregate bank run risk. If banks experience a shortfall of deposits, they borrow from the lender of last resort, CB at a penal rate. Excess reserve also earn an interest rate.

The government consumes some final goods which is financed by lump sum taxes on households and borrowing from the commercial banks and the CB via issuing long term government bonds. The CB finances its government bond holding by reserve creation, seigniorage and the revenue earned from banks resulting from penalty loans.

5.2 Households

Households solve the following maximization problem:

$$\max_{c_t, D_t, M_t^{TD}, H_t} E_t \sum_{t=0}^{\infty} \beta^t [U(c_t - \gamma_c C_{t-1}) + V(D_t/P_t) + W(M_t^{TD}/P_t) - \Phi(H_t)]$$

subject to the following budget constraint:

$$P_t c_t + D_t + M_t^{TD} \leq W_t H_t + (1 + i_t^D) D_{t-1} + M_{t-1}^{TD} + TR_t \tag{1}$$

where P_t is aggregate price index, c_t is the representative agent's consumption after adjusting for the previous period's aggregate consumption C_{t-1} up to a fraction γ_c which means a habit persistence relative to aggregate consumption. D_t is one period deposit in nominal terms which are perfect substitutes for short term government bonds (as in Gertler and Karadi, 2013), H_t is labour hours, W_t is nominal wage, i_t^D is the riskfree nominal interest rate on deposits, M_t^{TD} is transaction demand for cash, TR_t is lump sum transfer to the households which include cash flows from capital goods firms, retail goods firms, and banks as well as transfer from the government and cash injection from the CB. We assume that household receives direct utility from bank deposits and cash holding.[12] $U(.), V(.), W(.)$ are instantaneous continuous, strictly concave utility functions from consumption, real deposit and real money balance with the usual regularity conditions and $\Phi(H_t)$ is the continuous disutlity function from work.

The first order conditions are:

$$D_t : \; U_{ct} = V'(d_t) + \beta E_t U_{ct+1}(1 + i^D_{t+1})(P_t/P_{t+1}) \tag{2}$$

$$M^{TD}_t : \; U_{ct} = W'(m^{TD}_t) + \beta E_t U_{ct+1}(P_t/P_{t+1}) \tag{3}$$

$$H_t : \Phi'(H_t) = (W_t/P_t)\; U_{ct} \tag{4}$$

where $d_t = D_t/P_t$, $m^{TD}_t = M^{TD}_t/P_t$ and U_{ct} is the derivative of $U(c_t - \gamma_c C_{t-1})$ with respect to c_t. Equation (2) shows that marginal utility cost of holding a dollar of deposit balances the temporal marginal utility of liquidity service from deposits and the discounted utility benefits of the interest on deposit adjusted for inflation tax. Likewise equation (3) shows the marginal equivalence condition of cost and benefit of holding a dollar money balance. Equation (4) is the standard static efficiency condition for labour supply.

5.3 Capital goods producing firms

Capital goods producers buy last period's used capital from the wholesale firms/entrepreneurs $\{(1-\delta_k)\,K_{t-1}\}$ at real prices Q_t. They produce new capital stock K_t by investing I_t of final goods using a linear investment technology:

$$K_t = (1-\delta_k)K_{t-1} + Z_{xt}I_t \tag{5}$$

where Z_{xt} is an investment specific technology shock and δ_k is the physical rate of depreciation of capital. After investment this new capital is sold to the wholesalers at a price Q_t. For one unit investment, the capital goods producers purchases $(1 + S\left(\frac{I_t}{I_{t-1}}\right))$ of final goods where

[12] We put both real cash balance and real deposits in the utility function motivated by the fact that both money and short term bank deposits provide different kinds of transaction conveniences to the household. Putting real cash balance in the utility function has a long tradition following Sidrauski (1967). The idea of real deposits in the utility function is borrowed from Hansen and Imrohoroglu (2013) who put short term government bonds in the utility function. Since households value the liquidity service of short term bank deposits, they are willing to accept a lower rate on bank deposits than the loan rate the banks charge to the wholesale goods firms which are also owned by households. A natural borrowing-lending spread or limits to arbitrage thus arises in our model.

$S(.)$ is a continuous flow investment adjustment cost function.[13] The capital goods producer then solves

$$\max_{I_{t+j}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} \Pi^k_{t+j}$$

where, $S(1) = S'(1) = 0$ and $S''(1) = \kappa$ is the investment adjustment cost parameter and $\Omega_{t,t+s}$ is the inflation adjusted stochastic discount factor which is equal to $\frac{\beta^s U_{ct+s}}{U_{ct}}.\frac{P_t}{P_{t+s}}$. Π^k_{t+j} is the cash flow of the capital goods producer given by:

$$\Pi^k_{t+j} = P_{t+j}\left[Q_{t+j}I_{t+j} - \left\{1 + S\left(\frac{I_{t+j}}{I_{t+j-1}}\right)\right\} I_{t+j}\right]$$

The first order condition gives the following Euler equation:

$$Q_t = 1 + S\left(\frac{I_t}{I_{t-1}}\right) + S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - E_t \frac{\beta U_{ct+1}}{U_{ct}}\left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{6}$$

This Q equation is similar to Gertler and Karadi (2013).

5.4 Wholesale goods producing firms

Wholesale firms are run by risk neutral entrepreneurs who produce intermediate goods (Y^W_t) for the final goods producing retailers in a perfectly competitive environment. The entrepreneurs hire labour force from the households and purchase new capital from the capital good producing firms. They borrow L_t amount of loan from the bank in order to meet the value of new capital $(Q_t K_t)$. We assume that all capital spending is debt financed. Used capital at date t is sold at the resale market at the price Q_t.

Balance sheet condition of wholesale firms is:

$$Q_t K_t = \left(\frac{L_t}{P_t}\right) \tag{7}$$

Given $Y^W_t = A_t K^{\alpha}_{t-1} H^{1-\alpha}_t$ with $0 < \alpha < 1$ and A_t as the TFP shock, the equilibrium real wage, $W_t/P_t = (1-\alpha)\frac{(P^W_t/P_t)Y^W_t}{H_t}$ where P^W_t is the nominal price of the wholesale good.

[13] Note that this investment adjustment cost is incurred before investment is undertaken to install new capital K_t. That is why it does not appear in the linear investment technology (5).

The gross rate of return from capital is given by,

$$
\begin{aligned}
1+r_{t+1}^{k} &= \frac{(P_{t+1}^{W}/P_{t+1})Y_{t+1}^{W}-(W_{t+1}/P_{t+1})H_{t+1}+(1-\delta_{k})\,K_{t}Q_{t}}{Q_{t}K_{t}} \\
&= \frac{(P_{t+1}^{W}/P_{t+1})\left(\frac{Y_{t+1}^{W}}{K_{t}}\right)-(1-\alpha)\,\frac{(P_{t+1}^{W}/P_{t+1})Y_{t+1}^{W}}{H_{t+1}}\left(\frac{H_{t+1}}{K_{t}}\right)+(1-\delta_{k})\,Q_{t}}{Q_{t}} \\
&= \frac{(P_{t+1}^{W}/P_{t+1})MPK_{t+1}+(1-\delta_{k})Q_{t+1}}{Q_{t}}
\end{aligned}
$$

where MPK_{t+1} denotes the the marginal product of capital at date $t+1$. Defining i_{t}^{L} as the net nominal interest rate on loans, the optimality condition for firms demand for capital (or the arbitrage condition) can be written as,

$$
1+r_{t+1}^{k}=\left(1+i_{t+1}^{L}\right)\frac{P_{t}}{P_{t+1}}
$$

which yields,

$$
(1+i_{t+1}^{L})=\frac{P_{t+1}^{W}MPK_{t+1}+(1-\delta_{k})\,P_{t+1}Q_{t+1}}{P_{t}Q_{t}}
$$

In other words,

$$
1+i_{t+1}^{L}=\left[\left(\frac{P_{t+1}^{W}}{P_{t+1}}\right)\frac{MPK_{t+1}}{Q_{t+1}}+1-\delta_{k}\right]\left[\frac{P_{t+1}Q_{t+1}}{P_{t}Q_{t}}\right] \tag{8}
$$

5.5 Final goods retail firms

Retailers buy intermediate goods at a price P_{t}^{W}, convert it one-to-one to final goods, and differentiate the goods at a zero cost. The i^{th} retailer sells his/her unique variety of final product $y_{t}(i)$ applying a markup over the wholesale price after factoring in the market demand condition which is characterized by the price elasticity $\left(\varepsilon^{Y}\right)$. Retailer's prices are sticky and indexed to steady state inflation. They also face a quadratic price adjustment cost which is parameterized by ϕ_{P}. Hence, the ith retailer chooses $\{P_{t+j}\,(i)\}_{j=0}^{\infty}$ to maximize present value of their expected cash flows $(\Pi_{t+j|t}^{r}(i))$ conditional on the information at date t.

$$
\max_{\{P_{t}(i)\}} E_{t}\sum_{j=0}^{\infty}\Omega_{t,t+j}\left\{\Pi_{t+j|t}^{r}(i)\right\}
$$

subject to

$$y_{t+j|t}(i) = \left(\frac{P_{t+j}(i)}{P_{t+j}}\right)^{-\varepsilon^Y} y_{t+j}$$

where $y_{t+j|t}(i)$ is the demand for the i^{th} retailer's good conditional on the aggregate demand y_{t+j} and

$$\Pi^r_{t+j|t}(i) = P_{t+j}(i)\, y_{t+j}(i) - P^W_{t+j}(i)\, y^W_{t+j}(i) - \frac{\phi_p}{2}\left[\left\{\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - (1+\pi_{t+j-1})^{\theta_p}(1+\bar{\pi})^{1-\theta_p}\right\}^2 P_{t+j}y_{t+j}\right]$$

where $\phi_p > 0$ and $0 < \theta_p < 1$.

Note that θ_p is an indexation parameter. This price adjustment cost specification is borrowed from Gerali et al. (2010). The aggregate demand at date t (y_t) is given by a standard CES aggregator:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon^Y - 1}{\varepsilon^Y}}\, di\right]^{\frac{\varepsilon^Y}{\varepsilon^Y - 1}}; \qquad \varepsilon^Y > 1$$

The first order condition after imposing symmetric equilibrium yields

$$1 - \varepsilon^Y + \varepsilon^Y\left(\frac{P_t}{P^W_t}\right)^{-1} - \phi_p\left\{1+\pi_t - (1+\pi_{t-1})^{\theta_p}(1+\bar{\pi})^{1-\theta_p}\right\}$$

$$+\Omega_{t,,t+1}\phi_p\left\{1+\pi_{t+1} - (1+\pi_t)^{\theta_p}(1+\bar{\pi})^{1-\theta_p}\right\}\pi_{t+1}\frac{y_{t+1}}{y_t} = 0 \tag{9}$$

In the steady state, when $\pi_{t+1} = \pi_t = \pi$, the above price equation reduces to a simple static markup equation:

$$\frac{P}{P^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}. \tag{10}$$

5.6 The Banking sector

Denote L_{t-1} as outstanding loans issued at date $t-1$, and B^P_{t-1} as the corresponding outstanding net government bonds held by the commercial banks. Likewise, let M^{RD}_{t-1} be commercial banks' outstanding reserve holding at date $t-1$. Banks plan to hold excess reserve because they face the risk of a bank run at the end of period. As in Chang et al. (2014) at the end of each period, deposits can be withdrawn stochastically. If the withdrawal (say $\widetilde{W_{t-1}}$) exceeds bank reserve (cash in vault), banks fall back on the CB for emergency loan at a penalty rate i^p_t mandated by CB. Banks pay back to the CB at the end of the period. We assume that this withdrawal risk is an aggregate risk which cannot be mitigated through any interbank market.[14] This withdrawal uncertainty necessitates a demand for excess reserve by the banks even though the interest rate on excess reserve can become negative.[15]

Define i_t^R as the interest rate on excess reserve (IOER) mandated by BoJ, $\widetilde{W}_{t-1}$ =stochastic withdrawal at the end of period t and S_t is the date t price of a nominal default free long term bond with geometrically declining coupon payments at rate ν as in Rudebusch and Swanson (2012). Bank's cash flow at date t can be rewritten as:

$$\begin{aligned} CF_t^b &= (1+i_t^L)L_{t-1} + (1+i_t^R)M_{t-1}^{RD} + (1+\nu S_t)B_{t-1}^P - (1+i_t^D)D_{t-1} \\ &\quad - (1+i^p)D_{t-1}E\max(\widetilde{W}_{t-1}/D_{t-1} - M_{t-1}^{RD}/D_{t-1}, 0) \\ &\quad - S_t B_t^P - L_t - M_t^{RD} + D_t \end{aligned} \tag{11}$$

Given the assets at date t, and deposit sequence $\{D_t\}$ determined by the household's problem, banks choose M_t^{RD}, B_t^P, L_t which solves the following dynamic optimization:

$$\underset{M_t^{RD}, B_t^P, L_t}{\text{Max}} \quad E_t \sum_{s=0}^{\infty} \Omega_{t,t+s} CF_{t+s}^b$$

s.t. the statutory reserve requirement:

$$M_t^{RD} \geq \alpha_r D_t \tag{12}$$

The Euler equation for M_{t+1}^{RD} is given by:

$$M_t^{RD}: \; 1 = E_t \Omega_{t,t+1} \left[(1+i_t^R) + (1+i_t^p) \text{Prob}(\widetilde{W}_t/D_t \geq M_t^{RD}/D_t) \right] + \varkappa_t \tag{13}$$

The first term in the square bracket in (13) is the bank's interest income from reserve and the second term is the expected saving of penalty because of holding more reserve and $\varkappa_t$ is the Lagrange multiplier associated with the reserve constraint (12).

[14] We do not have any interbank market for loans in this model because banks are all homogenous and subject to the same aggregate risk of bank run. However, since a higher penalty rate induces banks to hold more excess reserve (which we show later), this rate is a reasonable proxy for the overnight call rate.

[15] Think it like this. The withdrawal is expected at the end of the period. Suppose the bank anticipates that it will fall short of reserve by 20 dollars at the end of the day. The CB charges 10 percent penalty rate. Thus the bank's expected penalty is 22 dollars which includes the principal and interest that the bank has to pay at the end of the period or the start of the next period. Taking this into consideration, bank chooses the reserve holding optimally at the start of date t. Thus a higher expected penalty will make the bank hold more cash reserve.

The Kuhn Tucker condition states that

$$\frac{M_t^{RD}}{D_t} = \alpha_r \text{ if } \varkappa_t > 0$$

Otherwise

$$E_t\Omega_{t,t+1}\left[(1+i_t^R) + (1+i_t^p)\text{Prob}(\widetilde{W}_t/D_t \geq M_t^{RD}/D_t)\right] = 1 \quad (14)$$

Assuming a uniform distribution for $\widetilde{W}_t$ over $[0, D_t]$, (14) reduces to:[16]

$$M_t^{RD} : 1 = E_t\Omega_{t,t+1}\left[(1+i_t^R) + (1+i_t^p)(1 - \frac{M_t^{RD}}{D_t})\right] \quad (15)$$

Solve $\frac{M_t^{RD}}{D_t}$ as follows:

$$\frac{M_t^{RD}}{D_t} = 1 - \frac{1-(1+i_t^R)E_t\Omega_{t,t+1}}{(1+i_t^p)E_t\Omega_{t,t+1}} \quad (16)$$

Since $(1+i_t^R)E_t\Omega_{t,t+1} < 1$, given the discount factor, $\Omega_{t,t+1}$, a higher i_t^R or i_t^p means a higher proportion of deposits held as reserve (M_t^{RD}/D_t) by the banks. It is straightforward to verify that at the steady state $\frac{\partial(M_t^R/D_t)}{\partial i_t^R} = \frac{1}{1+\bar{i^p}}$ and $\frac{\partial(M_t^R/D_t)}{\partial i_t^p} = \frac{1}{(1+\bar{i^p})^2}\left[\frac{1+\bar{\pi}-\beta(1+\bar{i^R})}{\beta}\right]$. For the baseline calibrated values of the steady state parameters reported later in Table 2, $\frac{\partial(M_t^R/D_t)}{\partial i_t^R} > \frac{\partial(M_t^R/D_t)}{\partial i_t^p}$. In other words, a decline in the IOER has a larger quantitative effect than a drop in the overnight borrowing rate on depressing the banks' excess reserve. Thus banks respond by loaning out excess reserve more with respect to a negative shock to IOER than a negative shock to overnight borrowing rate. This result is crucial in understanding later why a the recent QQE experiment of negative IOER is a more effective tool for promoting the economy than a CMP of lowering the overnight borrowing rate.

Next the bank solves a recursive problem of choosing B_t^P and L_t given B_{t-1}^P and L_{t-1} which were chosen in the previous period. This is a dynamic allocation problem. We have the two Euler equations:

$$B_t^P : 1 = E_t\Omega_{t,t+1}(1+\nu S_{t+1})/S_t \quad (17)$$

$$L_t : \ 1 = E_t\Omega_{t,t+1}(1+i_{t+1}^L) \quad (18)$$

Since $\Omega_{t,t+1}(s)$ is nothing but a contingent claims price of a dollar, above two equations

[16] We assume that the $\widetilde{W}_t$ cannot exceed the available deposit D_t which makes $\widetilde{W}_t/D_t$ bounded above by unity.

basically mean a no arbitrage condition that the discounted value of the expected excess returns on bond and loan is zero.

5.7 CB budget constraints

We now characterize the supply of bank reserve (denoted M_t^R) and supply of currency (denoted M_t^T). As in Hall and Reis (2015), CB must create enough reserve to pay for the interest and principal on existing commercial bank reserves, cover the purchase of government bonds (B_t^{CB}) net of bond income held from previous bonds (which is $(1+\nu S_t)B_{t-1}^{CB}$), and dividends that it pays to the government after netting out the penalty revenue that it receives from banks from overnight loans and the seigniorage revenue from printing cash ($M_t^T - M_{t-1}^T$). In other words, CB's budget constraint is given by:

$$M_t^R = (1+i_t^R)M_{t-1}^R + S_t B_t^{CB} - (M_t^T - M_{t-1}^T) - (1+\nu S_t)B_{t-1}^{CB} - (1+i_t^p)D_t E\max(\widetilde{W}_t/D_t - M_t^R/D_t, 0) + Div_t \tag{19}$$

Note that the idea of dividend payment by the CB to the government is borrowed from Hall and Reis (2015). Literally, the CB does not pay such dividend but it should generate sufficient revenue to cover the deficits of the government. Thus Div_t is the link between CB and the government.

$$Div_t = m_t^R - (1+i^R)\frac{m_{t-1}^R}{1+\pi_t} + m_t^T - \frac{m_{t-1}^T}{1+\pi_t} - S_t b_t^{CB} + (1+\nu S_t)\frac{b_{t-1}^{CB}}{1+\pi_t} + 0.5(1+i^p)d_t\left(1 - \frac{M_t^{RD}}{d_t}\right)^2 \tag{20}$$

where $b_t^{CB} = B_t^{CB}/P_t$, $m_t^B = M_t^B/P_t$, $m_t^T = M_t^T/P_t$.[17] The last term on the right hand side of (20) is obtained by assuming that $\widetilde{W}_t$ is drawn from a uniform distribution over $[0, D_t]$.

5.8 Government Budget Constraint

The government spends exogenous stream (G_t) of final goods. This spending is financed by lump sum taxes on households (T_t), and the dividends (Div_t) received from the CB. All government borrowing is in the form of long term government bonds (B_t^G). The government budget constraint in nominal form is given by:

$$P_t G_t + (1+\nu S_t)B_{t-1}^G = P_t T_t + S_t B_t^G + Div_t \tag{21}$$

[17] $S_t b_t^{CB}$ (which is $S_t B_t^{CB}\ /P_t$) is the real holding of government bonds.

5.9 Equilibrium

In equilibrium, the following market clearing conditions hold:

1. Goods market clears which means that GDP equals the sum of consumption, private investment, government spending, and price adjustment costs.

$$c_t + \left\{1 + S\left(\frac{I_t}{I_{t-1}}\right)\right\} I_t + G_t + \frac{\phi_p}{2}\left[\left\{(1+\pi_t) - (1+\pi_{t-1})^{\theta_p}(1+\bar{\pi})^{1-\theta_p}\right\}^2 y_t\right] = y_t$$

2. The loan market clears in the sense that the balance sheet constraint (7) of the wholesaler binds:[18]

$$L_t/P_t = Q_t K_t$$

3. Given that all public debt is nationally held, the bond market equilibrium requires that JGB held by banks and the CB sum to the treasury issued bonds

$$B_t^P + B_t^{CB} = B_t^G$$

4. Money market clears which means that the demand for bank reserve (M_t^{RD}) equals the supply bank reserve (M_t^R) and the transaction demand for money (M_t^{TD}) equals the corresponding supply (M_t^T):

$$M_t^{RD} = M_t^R$$

$$M_t^{TD} = M_t^T$$

5.10 Debt/GDP target

The value of the public debt is $S_t B_t^G$. We assume that the debt to GDP ratio is exogenously fixed at Γ. In other words,

$$S_t B_t^G = \Gamma P_t y_t$$

Given that S_t and y_t are stationary, the government has to issue nominal debts at the target rate of inflation ($\bar{\pi}$).

5.11 QE operation

In practical terms, a QQE is analogous to an open market operation where the CB is buying JGBs from the banks in the short run and place more reserve in the banking system. This

[18] An endgenous credit rationing emerges in a steady state equilibrium because of a positive borrowing-lending spread which we discuss in the section 5.14.1. Thus loan market does not clear in the usual sense that loan demand is equal to deposit.

alters the composition of assets (reserve/bond ratio) of the commercial banks in the short run. The commercial banks have a higher proportion of bank reserve and lower proportion of government bonds following the QQE operation. Since it is a short run policy exercise, such an operation should not permanently alter the asset composition of the CB. In addition, in the context of Japanese QQE operation, a two percent inflation target is also taken into consideration by the CB while undertaking the QQE operation.

Keeping these features in mind, we formulate the QE operation as a stochastic shock to monetary reserve and the ratio of JGBs held by CB (B_t^{CB}/B_t^G) as follows: [19]

$$\frac{M_t^R/M_{t-1}^R}{1+\bar{\pi}} = \left(\frac{M_{t-1}^R/M_{t-2}^R}{1+\bar{\pi}}\right)^{\rho_\mu} \exp(\xi_t^\mu) \tag{22}$$

$$(\lambda_t/\bar{\lambda}) = (\lambda_{t-1}/\bar{\lambda})^{\rho_\mu} \ \exp(\xi_t^\mu) \tag{23}$$

where $\lambda_t = B_t^{CB}/B_t^G$ and ξ_t^μ is an iid QE shock. Notice that the same shock, ξ_t^μ appear in both the monetary base and the asset composition equations (22) and (23). A positive QE shock (ξ_t^μ) boosts the monetary base and raises the ratio of bank reserve in the monetary base $M_t^R/(M_t^R + M_t^T)$. It also raises the share of CB holding of government bonds, λ_t in the short run. This feature reflects the basic tenets of QE operation that the CB purchases government bonds from commercial banks by injecting bank reserve which changes the asset composition of the CB and commercial banks. In the long run monetary base continues to grow at the target inflation rate and CB and commercial bank's asset composition reverts to the steady state ($\bar{\lambda}$) which is calibrated in the model. The rates of convergence of M_t^R/M_{t-1}^R and λ_t to the respective steady states are assumed to be the same which explains why the same smoothing coefficient, ρ_μ appears in both (22) and (23).

This law of motion (22) for the monetary base means that in a deterministic steady state the monetary base grows at the target rate of inflation $\bar{\pi}$. Such a money supply process imposes restriction on the short run growth rate of real reserve (m_t^R) and inflation as follows:

$$\frac{(1+\pi_t)(m_t^R/m_{t-1}^R)}{1+\bar{\pi}} = \left(\frac{(1+\pi_{t-1})(m_{t-1}^R/m_{t-2}^R)}{1+\bar{\pi}}\right)^{\rho_\mu} \exp(\xi_t^\mu) \tag{24}$$

where ξ_t^μ is an iid shock to the monetary base with zero mean.

What is the implication of such QQE for the balance sheets of BoJ and commercial banks? Note that when BoJ buys JGBs, it is buying debt of the government from the

[19] Such an operation can also be thought of as a Repo operation by the CB. In other words, the CB is making a collateralized loans to the commercial banks.

commercial banks. Thus the BoJ augments its asset by holding more government bonds and it simultaneously creates more liability by increasing monetary base. Thus the CB's balance sheet grows. Commercial Bank's assets just undergoes a maturity transformation from long term bonds to equivalent short term bank reserves.

Since real reserve is proportional to deposit as shown in the bank's reserve demand function, it also imposes restriction on the dynamics of deposits, interest rate on loans and consumption.

5.12 Policy Rates

There are two key policy rates: (i) the penal rate i_t^p and (ii) the IOER, i_t^R. A reasonable proxy for i_t^p is the overnight borrowing rate although we do not have an interbank market for loan. The official discount rate may be another proxy for i_t^p. However, it does not change much over time in response to macroeconomic condition. One can thus interpret i_t^p as a policy rate that would have prevailed if BoJ had used interest rate as a policy tool instead of quantitative easing. Viewed from this perspective, one can interpret i_t^p as a shadow interest rate in line with Iwasaki and Sudo (2017). We assume the following standard Taylor rule that characterizes the law of motion for i_t^p:

$$\frac{1+i_t^p}{1+\overline{i^p}} = \left(\frac{1+i_{t-1}^p}{1+\overline{i^p}}\right)^{\rho_{i^p}} \left(\frac{1+\pi_{t-1}}{1+\bar{\pi}}\right)^{\phi_\pi(1-\rho_{i^p})} \exp(\xi_t^{ip}) \tag{25}$$

where ρ_{i^p} is the smoothing coefficient and ϕ_π is the inflation sensitivity of the interest rate, $\bar{\pi}$ is the target inflation rate and ξ_t^{ip} is an iid shock to the policy rate, i_t^p with zero mean.

The law of motion for IOER is posited as follows:

$$1+i_t^R = \left(1+i_{t-1}^R\right)^{\rho_{iR}} \exp(\xi_t^{iR}) \tag{26}$$

where ρ_{iR} is the smoothing coefficient and ξ_t^{iR} is an iid shock to the IOER with a zero mean. The underlying assumption is that IOER gravitates to a zero rate in the steady state.

5.13 Forcing Processes

We assume the following specifications for the three forcing processes, namely TFP (A_t), an investment specific technology (IST) shock (Z_{xt}), the fiscal spending shock (G_t):

$$A_t = \overline{A}^{1-\rho_A} A_{t-1}^{\rho_A} \xi_t^A$$

$$Z_{xt} = \overline{Z_x}^{1-\rho_z} Z_{t-1}^{\rho_z} \ \xi_t^z$$

$$G_t = \overline{G}^{1-\rho_G} G_{t-1}^{\rho_G} \ \xi_t^G$$

The monetary policy block is summarized by three equations, namely (24), (25) and (26). Since monetary base and policy rates were used in different regimes quite randomly, we shock one of these three policy equations at a time. This justifies the assumption that $\{\xi_t^{\mu}\}$, $\{\xi_t^{ip}\}$ and $\{\xi_t^{iR}\}$.

5.14 Interest rates and bond yields

5.14.1 Borrowing-lending spread

A steady state borrowing-lending spread emerges in this model because deposit appears in the utility function and provides transaction convenience to the household. Thus the household is willing to accept a lower interest on its bank deposit than the loan rate. This means an endogenous credit rationing emerges in the model in the steady state because the borrowing-lending spread is positive. To see it combine (2) and (18) to get the following steady state borrowing-lending spread:

$$i^L - i^D = \frac{(1+\pi)}{\beta} \frac{V'(d)}{U'(c)} > 0 \tag{27}$$

5.14.2 Long term bond yield and term premium

Following Rudebusch and Swanson (2012), we focus on two important bond yield variables, (i) the yield to maturity and (ii) the term premium. Long term bond price (S_t) is modelled as perpetual consol type with geometrically declining coupon payments. Such a bond pricing equation obeys the Euler equation (17). The parameter ν pins down the duration of such a bond. We assume that these nominal long term bonds are default free and can be bought and sold every period.

A continuously compounded nominal yield to maturity ($nytm$) of such a long term bond is given by the following formula as in Rudebusch and Swanson (2008)[20]

$$nytm_t = \ln\left(\frac{1+\nu S_t}{S_t}\right) \tag{28}$$

The real yield to maturity ($rytm_t$) is defined as $nytm_t$ minus the rate of inflation (π_t). Note that S_t fluctuates as the stochastic discount factor $\Omega_{t,t+1}$ shows fluctuations that reflect inflation and consumption uncertainty.

Following Rudebusch and Swanson (2008), we define the term premium on a long term bond as the difference between yield on the bond and the unobserved risk neutral yield on the same bond. The risk neutral counterpart of (17) can be written as:

$$S_t^R = \beta E_t \frac{[1+\nu S_{t+1}^R]}{[1+\pi_{t+1}]}$$

The term premium (tp) is thus defined as:

$$tp_t = \ln\left(\frac{1+\nu S_t}{S_t}\right) - \ln\left(\frac{1+\nu S_t^R}{S_t^R}\right) \tag{29}$$

Note that the term premium is a measure of risk of long term bonds. It is larger if the absolute value of the covariance between stochastic discount factor ($\Omega_{t,t+1}$) and the price of bond is higher.[21] Since the covariance depends on the inflation and the consumption growth, the term premium represents inflation and consumption risk associated with the long term bond.

The nominal one period holding period return of bond with maturity n for any investor sitting at date t as:[22]

$$nhpr_{t,t+1} = \frac{1+\nu S_{t+1}}{S_t} \tag{30}$$

The real holding period return ($rhpr_{t,t+1}^n$) is $nhpr_{t,t+1}$ minus the corresponding gross rate of inflation.

[20] To get this formula, write (17) in continuous time as:

$$S_t = e^{-nytm_t} + e^{-nytm_t}\nu S_t$$

where $e^{-nytm_t} = \Omega_{t,t+1}$. This can be rewritten as (28). Our expression of $nytm$ differs from Rudebusch and Swanson (2008) equation 15 because they do not discount the first period coupon payment while we do because the first coupon payment comes after a period in our formulation.

[21] See Rudebush and Swanson (2008).

[22] To see this, note that a perpetual bond bought at date t at the price S_t has a resale value νS_{t+1} at date $t+1$ which explains the numerator term. Thus the total proceeds next period including the coupon payment is $1+\nu S_{t+1}$.

6 Estimation and Simulation

6.1 Baseline Calibration

In this section, we report the results of some policy experiments with our estimated DSGE model. For the baseline model, we fix the standard deep parameters β and α at the conventional levels of 0.99 and 0.33 respectively. The capital depreciation rate is fixed at a conventional level of 0.025. A long run 2% inflation target is set which means the steady state $\pi = .02$. The penalty interest rate $i^p = 0.001$ based on the quarterly average call rate. The steady state IOER, $i^R = 0$ and the required reserve ratio around 0.0067 (the average of all existing reserve ratios in Japan). For these baseline values, we find that the marginal benefit of holding an extra excess reserve exceeds the cost. Thus it is reasonable to assume that all banks start off with an excess reserve meaning that the Lagrange multiplier $\varkappa_t = 0$ in the steady state. The steady state proportion of government bonds held by BoJ ($\bar{\lambda}$) is fixed at 0.4.[23] The long run debt/GDP ratio (Γ) is fixed at 2.32.[24] The bond decay parameter ν is fixed at 0.975 to set the duration of these perpetual bonds close to 40 quarters. The interest rate on bank deposits is fixed at 4 basis points based on an average of short term ordinary deposits over our sample period 1999Q1-2017Q1. We specialize our simulation to a utility function: $\ln(c_t - \gamma_c C_{t-1}) + \eta_1 \ln d_t + \eta_2 \ln m_t^T - H_t$ and quadratic investment adjustment cost function: $S(I_t/I_{t-1}) = \frac{\kappa}{2}(I_t/I_{t-1} - 1)^2$. The bank deposit preference parameter (η_1) is fixed at 0.0435 based on the steady state deposit/consumption ratio of 1.5 available for the year 2017Q1. The cash preference parameter η_2 is fixed at 0.0318 based on the cash/consumption ratio of 1.08 over our sample period. Table 2 presents the baseline calibrated parameter values.

Table 1: Baseline Parameter Values

β	η_1	η_2	α	δ_k	$\bar{\lambda}$	ν	$\bar{\pi}$	Γ	$\bar{i}^r$	α^r	$\bar{i}_D$	$\bar{i}^p$
0.99	0.0435	0.0318	0.36	0.025	0.42	0.975	.02	2.32	0	.0067	0.0004	.001

[23] According to the Nikkei News on February 8th 2017, the JGB holding by BoJ is 358.1977 trillion yen at the end of January. At the end of January, the total amount outstanding of JGB is 894.3357 trillion yen. So the ratio is 40.05% which is a reasonable apprximation of $\bar{\lambda}$.

[24] On May 31st 2017 the Japanese government set a new fiscal policy target and said that they would try to decrease the debt/GDP ratio. The debt/GDP ratio is 232.4% at the end of 2016. Unless GDP increases or debt decreases, it is hard to decrease the debt/GDP ratio. Thus we fix 232.4% as a reasonable target for the debt/GDP ratio.

6.2 Bayesian Estimation

We undertake a Bayesian estimation to compute the remaining parameters for which there is less conventional wisdom. Since there are six forcing processes, we choose six observable for our estimation. Four key macroeconomic variables are chosen as observable namely (i) the real per capita GDP (y), (ii) The real per capita private nonresidential investment (i), (iii) the real per capita government consumption (G), (iv) the annualized CPI inflation (π). The remaining two variables are: (v) the annualized overnight call rate as a proxy for i_p (vi) the annualized nominal holding period return ($nhpr$). Choice of observable is based on a trial and error process and these six observable give the best fit of the model. The sample period is 1999Q1-2017Q1. Details of the data sources are explained in the appendix. First three level macroeconomic variables, y, i and G are demeaned by passing them through a one sided HP filter to make it comparable to the model.

Our selection of the probability density functions for the priors are based on educated guesses and available estimates from extant studies. For prior, the Beta distribution is used for the fractions while the Inverse Gamma distribution is specified for the parameters with non-negativity constraints.[25] The joint posterior distribution of the estimated parameters is obtained by standard procedure. First, the model equation system is loglinearized around the deterministic steady states and written in a linear rational expectation recursive form. Second, the system of equations is written in a Kalman filter observation equation form. Third, using this observation equation, the loglikelihood function of the relevant parameter vector is constructed. Fourth, the log posterior kernel is expressed using the prior density of the parameter. Fifth, the mode of this posterior kernel is computed using standard numerical optimization routines. Finally, a Gaussian approximation is constructed in the neighbourhood of this posterior mode using the Markov Chain Monte Carlo-Metropolis-Hastings (MCMC-MH) algorithm. This algorithm simulates the smoothed histogram that approximates the posterior distributions of parameters of our interest. Five parallel chains are used in the MCMC-MH with a 30% acceptance rate from the draws. The univariate and multivariate diagnostic statistics are computed to check for MCMC convergence. All the computations are done by using dynare 4.5.3 version.

Table 2 reports the baseline parameter estimates from the Bayesian estimation routine. Priors are fixed at conventional levels. The price adjustment cost coefficient is taken from Keen and Wang (2005). The prior for the investment adjustment cost coefficient κ is chosen

[25] The inverse gamma distribution of a random variable x with two parameters of υ and ϱ has $E(x) = \varrho/(\upsilon-1)$ and $Var(x) = \varrho^2/((\upsilon-1)^2(\upsilon-2))$. Setting the prior standard deviation to infinity means $\upsilon = 2$ and $\varrho = E(x)$. The rationale for setting the prior standard deviation to infinity is that it imposes no bound on the relevant non-negative estimable parameter and let the data determine its value using Bayesian updating.

following Fueki et al. (2016) . The posterior means of estimated parameters are reported with 90% highest posterior density (HPD) intervals. In most cases, data provide a lot of information about the parameters demonstrated by a significant difference between prior and posterior estimates. The only exceptions are for θ_p and ε_Y. The posterior estimate of the inflation targeting Taylor rule coefficient (ϕ_π) is in line with Fueki et al. (2016) while the investment adjustment cost parameter κ is a lot higher than their estimate. All our estimable parameters are properly identified in the model following the criteria of asymptotic information matrix and collinearity patterns of the parameters as suggested in Iskrev (2010a, b) and Iskrev and Rotto (2010a, b). This is summarized by the information matrix plotted in Figure 10.

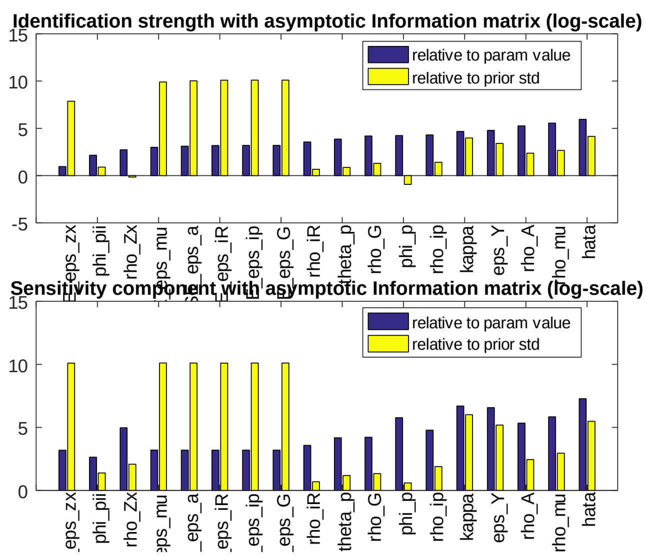

Figure 10: Identification Diagnostics

Table 3 gives a summary description of how the model matches the basic business cycle statistics. Since all level variables are one sided HP filtered, by construction they are demeaned and therefore, have a mean zero.[26] The model has mixed performance in matching standard deviations of real and financial macro aggregates. While it does really well in matching standard deviation of consumption and investment, it overstates the volatility GDP. On the bond market front, volatility front the model mostly underestimates the volatility of yields.

[26] Since it is a non-linear exact model, we follow Pfeier (2017) in specifying the observation equations. For example, denote a generic level variable as z_t. The observation equation is then written as: $z_t^{obs} = \ln z_t - \ln \bar{z}$ where $\bar{z}$ is the steady state of z_t. The right hand side is the demeaned log level of the model variable which is directly comparable to the one sided HP filtered variable in the data.

Table 2: Prior Densities and Posterior Estimates for Baseline Model

Parameters	Prior mean	Posterior Mean	90% HPD interval	Distribution	Prior sd
ρ_A	0.9	0.70	(0.56, 0.83)	beta	0.05
ρ_{zx}	0.90	0.80	(075, 0.85)	beta	0.05
ρ_G	0.90	0.88	(0.82, 0.93)	beta	0.05
ρ_μ	0.90	0.88	(0.82,0.98)	beta	0.05
ρ_{iR}	0.90	0.40	(0.39,0.42)	beta	0.05
ρ_{ip}	0.90	0.99	(0.99,0.99)	beta	0.05
ϕ_π	3.50	1.38	(0.81,1.93)	invgamma	inf
ϕ_p	176.0	186.83	(174.76, 195.18)	invgamma	inf
θ_p	0.20	0.21	(0.20, 0.22)	beta	0.01
γ_c	0.60	0.87	(0.85,0.90)	beta	0.10
ε_Y	4.00	4.00	(3.99, 4.00)	invgamma	inf
κ	2.0	52.29	(46.39, 58.96)	invgamma	inf
σ_A	0.001	0.78	(0.56,1.00)	invgamma	inf
σ_{zx}	0.001	3.12	(2.07,3.99)	invgamma	inf
σ_G	0.001	0.01	(0.01, 0.01)	invgamma	inf
σ_μ	0.001	0.03	(0.01, 0.04)	invgamma	inf
σ_{iR}	0.001	2.70	(2.34,3.05)	invgamma	inf
σ_{ip}	0.001	0.001	(0.001,0.001)	invgamma	inf

Table 3: Summary Statistics: Model vs Data

Variables	Mean (Model)	Mean (Data)	St dev (Model)	St dev (Data)
y	0	0	.0204	.012
c	0	0	.0154	.01
i	0	0	.0629	.05
$nytm$	.011	.012	.0002	0.005
$rytm$	.003	.012	.0023	.026
$nhpr$	.005	.02	.0005	.076
$rhpr$	.005	.02	.0020	.082
π	.005	0.00	.0025	.02
tp	0	.010	.0005	.005
i^L	.0303	.015	.0082	.001

6.3 Variance Decomposition

Table 4 shows the variance decompositions of fundamental shocks based on the average of all parameter draws.[27] Several points are in order. First, government spending shock and the overnight call rate have no contribution to any real and financial variables. Second, TFP shock accounts for less variation of the key real macroeconomic variables compared with monetary base shock except for employment. Third, on the bond market front, the nominal yield to maturity and nominal holding period returns are primarily influenced by the shocks to IOER while the real yields and real holding period returns are driven significantly by monetary base shock because of its effect on inflation. Finally, the term premium is driven by multiple shocks to the economy while monetary base shock still dominates.

Table 4: Posterior Mean Variance Decomposition

	ε^A	ε^G	ε^μ	ε^{zx}	ε^{ip}	ε^{iR}
y	33.76	0.00	46.41	19.11	0.00	0.72
i	13.46	0.00	14.68	71.85	0.00	0.01
c	17.10	0.00	24.08	58.21	0.00	0.61
h	85.93	0.00	4.84	9.13	0.00	0.10
π	38.84	0.00	49.36	11.63	0.00	0.17
$nytm$	0.00	0.00	27.31	0.00	0.00	72.69
$rytm$	40.91	0.00	45.39	12.30	0.00	1.41
$nhpr$	0.00	0.00	2.48	0.00	0.00	97.52
$\not{r}hpr$	23.23	0.00	19.88	6.93	0.00	49.96
tp	18.73	0.00	45.18	25.45	0.00	10.63

6.4 Parameter Stability

Since there were several episodes of monetary policy changes since the beginning of millennium, one may be concerned about parameter stability of the model across different regimes. Due to paucity of data, it is difficult to estimate our model for each policy regime. We take 2006Q2 as a cut off in view of a discrete monetary policy change that happened after 2006Q2. We thus split the sample in two parts (1999:Q1-2006:Q2) and (2006:Q3-2017:Q1). Table 4 presents the estimates of the key structural parameters. None of the second moment parameters of the underlying forcing processes show any remarkable shifts. Among the deep parameters, except the investment adjustment cost parameter (κ), all parameters show

[27] The variance decomposition is based on various parameter draws and based on the average of the posterior means over all five Markov chains.

reasonable stability. These estimates should be interpreted with caution because of limited number of observations during these two subsamples.

Table 5: Comapring QE and Post QE Estimates

Parameters	1999:Q1-2006:Q2	2006:Q3-2017:Q1
ρ_A	0.97	0.89
ρ_{zx}	0.91	0.86
ρ_G	0.88	0.85
ρ_μ	0.92	0.92
ρ_{iR}	0.43	0.42
ρ_{ip}	0.99	0.99
ϕ_π	1.29	1.29
ϕ_p	182.19	168.11
θ_p	0.20	0.18
γ_c	0.88	0.78
ε_Y	4.00	4.00
κ	26.08	46.92
σ_A	0.35	0.46
σ_{zx}	1.25	1.83
σ_G	0.01	0.01
σ_μ	0.02	0.02
σ_{iR}	3.13	2.45
σ_{ip}	0.001	0.001

6.5 Policy Simulations

We do policy experiments keeping in mind the five episodes of Japanese monetary policy since the inception of QE in 2001. First we look at the effect of a positive innovation ξ_t^μ to the monetary base equation (22) and the bond share equation (23). A positive ξ_t^μ means that the CB is injecting bank reserve to the private sector and the concomitant increase in λ_t means a larger holding of government bonds by the CB. This closely mimics a QE operation because such an operation entails CB purchase of JGB by open market operation. Since the growth rate of monetary base is a strictly mean reverting process (with a convergence to a 2% growth rate), it means that such a QE operation is temporary and it is phased out over time until the economy reverts to its steady state with a 2% target inflation. This policy experiment closely reflects the QE regime (i) and the onset of the QE tapering phase (ii).

From phase (ii), BoJ changed the operating from the outstanding balance of current accounts at the Bank to the uncollateralized overnight call rate. In terms of our stylized model, such an operation may be formulated by giving up the open market purchase of government bonds and using the interest rate on overnight loans (i_t^p) as the control instrument. We, therefore, examine how a shock to i_t^p which is governed by a Taylor type rule impacts the aggregate economy.

From phase (iii), the focus is shifted more on the interest rate on bank reserve as the policy instrument. We, therefore, ask how a shock to i_t^R impacts the aggregate economy.

6.5.1 Phase 1: Effect of a QE Shock

Figure 11 of impulse response functions (irf) summarize the results of the phase 1 QE experiment. A positive one standard deviation shock to the monetary base growth (ξ_t^μ) immediately translates into a positive inflation shock via the money supply rule (22). Higher inflation raises the real marginal cost via the staggered price adjustment cost equation (9) as in any standard new-Keynesian model which means P_t^w/P_t rises. Higher real marginal cost makes the value of the marginal product of capital and labour shift out which means wholesale firms buy more capital and hire more labour. This translates into a higher price of capital Q_t. Nominal interest rate on loan rises due to two reasons: (i) higher inflationary expectation (Fisher effect) and (ii) greater demand for loan. This resembles a Tobin effect of inflation on investment. Retail output supply also rises along the standard new Keynesian channel as real marginal cost rises. Higher real wage encourages workers to supply more labour. A wealth effect promotes consumption. Higher GDP boosts the transaction demand for currency by households. On the banking front, the ratio of reserve to deposit falls because a higher anticipated inflation imposes a tax on holding reserve. Banks thus advance more loans which are reflected by higher investment and higher Q.

On the bond market front, several things happen. Because of a spark in inflationary expectations, the nominal bond price declines which reflects a decline in the stochastic discount factor (sdf as shown in Figure 11). Consequently, nominal yield to maturity of 10 year bonds rises but quite insignificantly.[28] On the other hand, the real yield to maturity falls sharply which is due to higher inflation. Similar responses are also seen for nominal and real holding period returns. The term premium falls because of rising consumption growth which lowers the long bonds more than the short bonds. This suggests that a positive QE shock lowers the risk from long term bond holding. Overall effects of a positive QE shock are expansionary, output, investment, consumption and employment rise which accord well with the stylized facts.

[28] The effect on alternative maturity bonds is similar. Effectively a positive QE makes the yield curve shift outward.

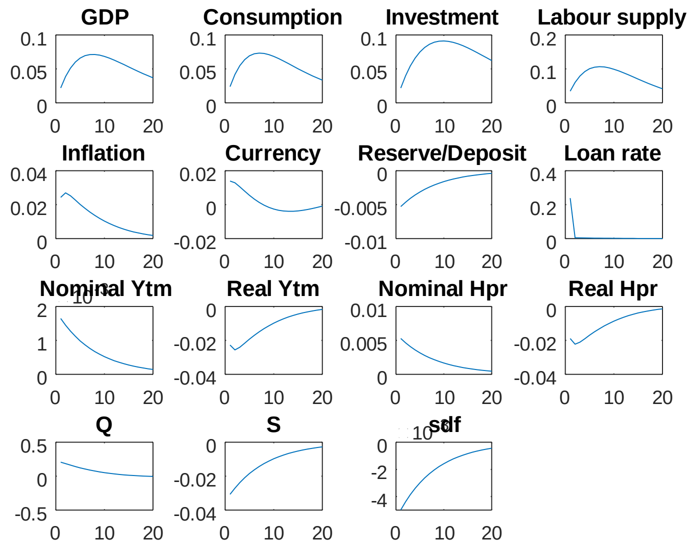

Figure 11: Macroeconomic Effects of a Positive QE Shock

6.5.2 Phase 2: Effect of a negative shock to the overnight borrowing rate

We next report the phase 2 policy simulation where the CB abandons reserve balance as an operating target and switches to the overnight call rate as the control instrument. Such a policy experiment is akin to a conventional monetary policy based on a Taylor rule (CMP). Figure 12 plots the effects of a negative shock to the overnight call rate which is deemed to be an easy money policy by the CB. A one standard deviation negative shock to overnight call rate lowers excess reserves of banks reflected by a decline in reserve/deposit ratio. Since more loan is released, investment rises which results in a higher Q. Higher Q also drives up the rental price of capital which via raising the real marginal cost of production makes the economy inflationary through the price adjustment equation (9). Consumption, investment and employment respond positively.

In contrast with a positive QE shock, on the bond market front both the nominal yield to maturity and nominal holding period returns declines marginally. This decline is due to the fact that the increase in consumption growth outweighs the rise in inflation which makes the real yield decline more than inflation. The term premium also decline. However, all these effects are rather miniscule in nature.

The overall effects on the macro economy are stimulative and similar to a positive QE shock. However, the effects are significantly weaker than a QE shock.

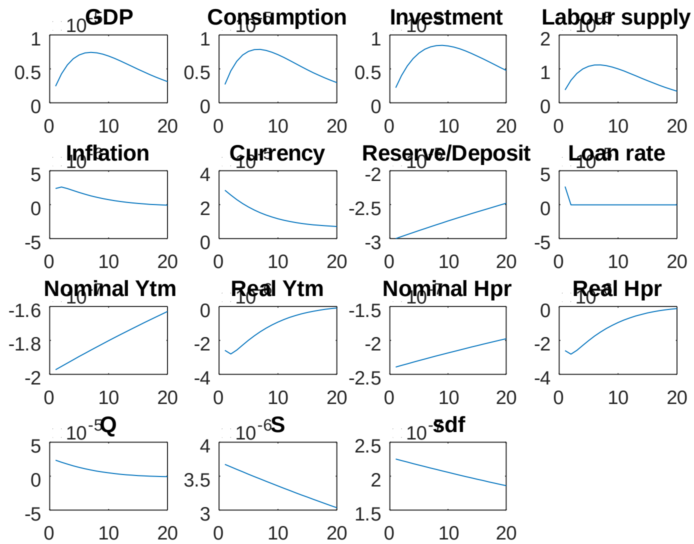

Figure 12: Macroecnomic Effects of a Negative Shock to the Overnight Borrowing Rate

6.5.3 Phase 3: Effect of a negative IOER

We now turn to the phase 3 policy simulation. What is the effect of a negative shock to IOER starting from a zero baseline rate? The effects and transmission mechanisms are quite similar to a drop in overnight borrowing rate shown in Figure 13 . However, the effect of a drop in IOER is considerably stronger than the reduction in the overnight borrowing rate. The intuition for this stems from the comparative statics properties of the banks' excess reserve demand function noted in section 5.6. Banks respond more by loaning out excess reserve to a negative shock to IOER than an equivalent reduction in the overnight borrowing rate.

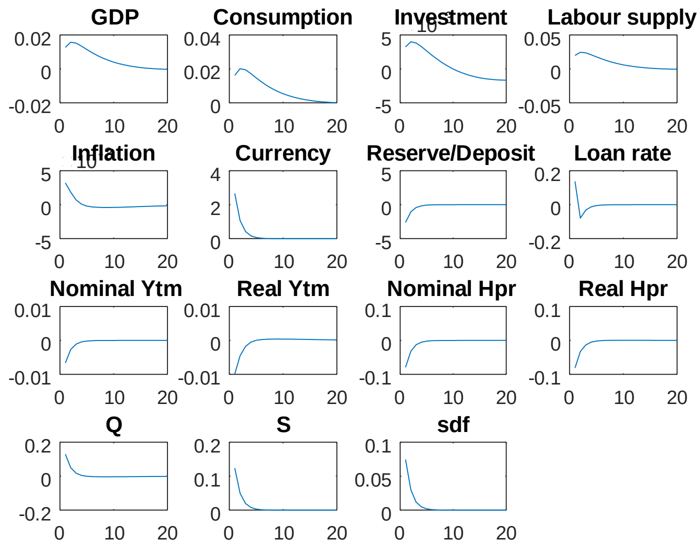

Figure 13: Macroeconomic Effects of a Negative IOER Shock

6.5.4 Term premia

Although alternative monetary policy shocks have different effects on the nominal yield to maturity, its effects on the term premia are rather similar. Since the duration of these perpetual bonds is positively related to the decay parameter ν, by varying ν we can conveniently compute the term premia (29) of bonds of different durations. Since the term premium picks up the consumption risk and is related to the covariance between marginal utility of consumption and asset excess returns, a second order approximation of the model is used to compute the term premium formula (29). Figures 14 through 16 summarize the effects of three monetary policy shocks on term premia of bonds of four durations, namely 5, 10, 20 and 30 years. The term premia uniformly decline for all these four bonds with the maximum effect on the shortest duration bonds (5 years). Not surprisingly the conventional overnight borrowing rate has near zero effects on the bond term premia for all durations. The effect of a negative IOER shock is largest on bonds of duration of five years.

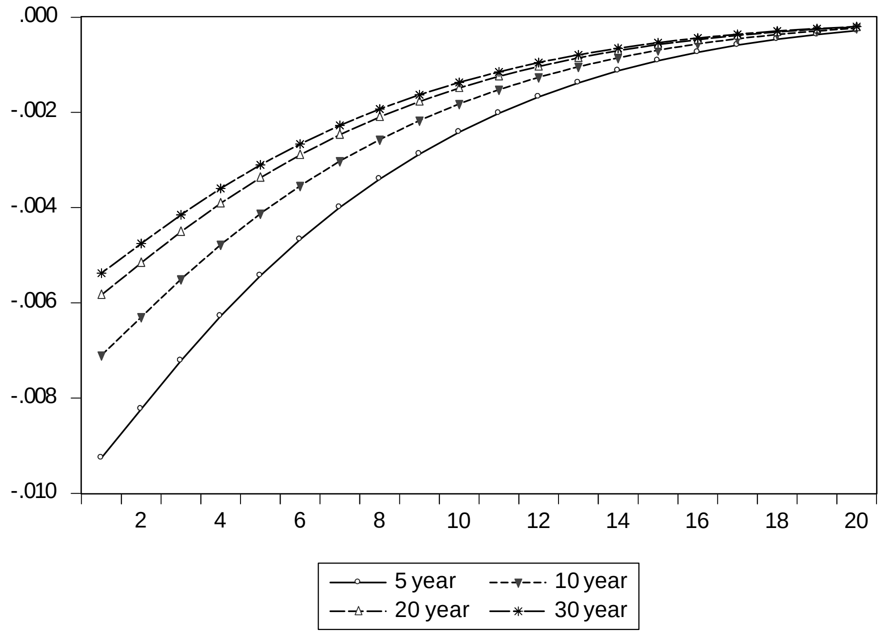

Figure 14: Effect of QE Shocks on Term Premia

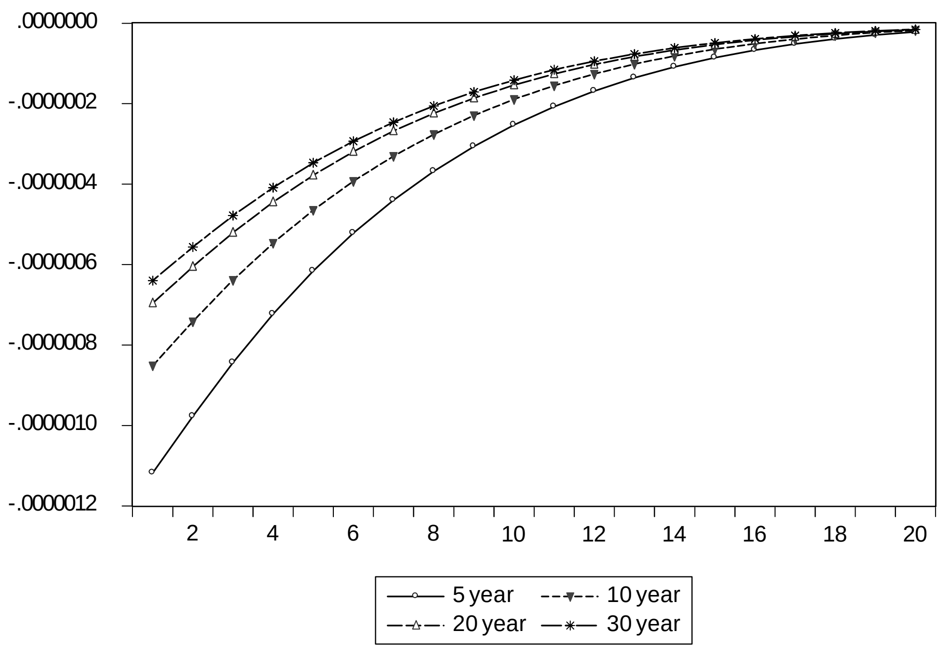

Figure 15: Effect of a Negative Shock to the Overnight Borrowing Rate on Term Premia

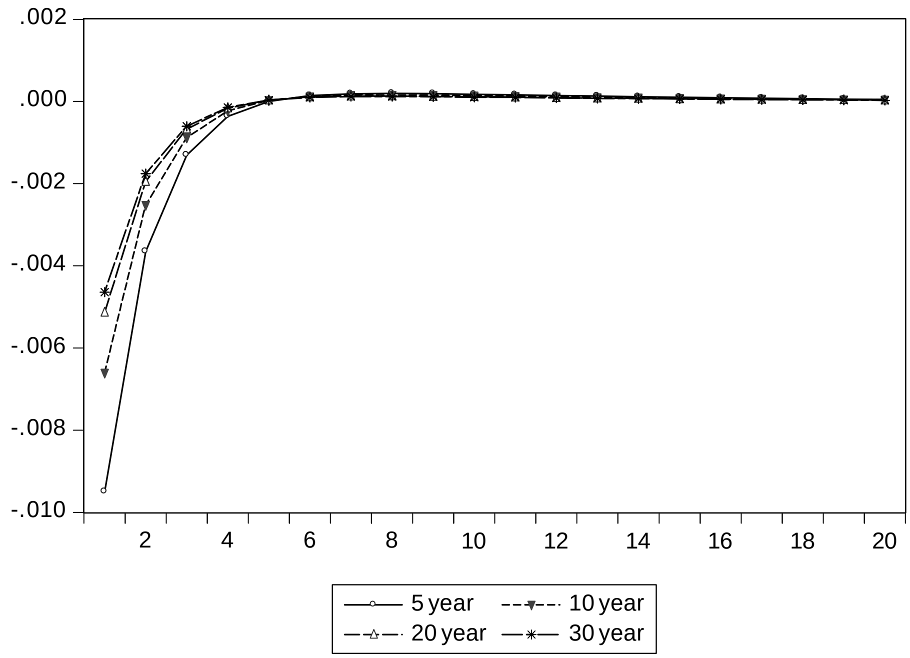

Figure 16: Effect of a Negative IOER Shock on Term Premia

6.6 Lessons for Yield Curve Control

The basic tenets of yield curve control by BoJ since September 2016 consist of two elements: (i) keep the nominal long term and short term bond yield at a low level, and (ii) let the inflation stay at least above two percent. A short run analysis with our DSGE model predicts that these two goals may not be attainable using a standard quantitative easing policy. The irf plots as seen in Figure 11 show that a positive QE shock raises inflation and the nominal yield to maturity simultaneously. This happens due to the fact that a positive inflationary expectation resulting from a QE shock raises all nominal rates including the yield to maturity. The real yield to maturity, however, falls but it is not enough to outweigh the inflationary expectation ignited by QE. The recent QQE experiment of lowering the IOER is, however, encouraging in this context. As seen in the irf charts in Figures 13 and 14, a negative IOER shock lowers the nominal yield to maturity but it also raises inflation making both goals attainable at least in the short run.

The contradiction between two policy goals is sharpened in the long run. This can be easily seen by noting that in the long run the basic arbitrage condition dictates that the loan rate and the bond yield are equal and proportional to the inflation rate meaning (see equations (17) and (18))

$$1 + i_L = \frac{1 + \nu S}{S} = \frac{1 + \pi}{\beta} \tag{31}$$

Note that this is the basic Fisher's relationship between nominal interest rate and inflation which prevails in the long run. The immediate implication is that a yield curve control and inflation targeting are mutually contradictory long run goals. A simple back-of-the-envelope calculation based on the long run arbitrage condition (31) indicates that a long run inflation target of 2% means a long run yield of 3.03% (given $\beta = .99$) which is far away from near zero yield target.

The lesson for the policy of yield curve control is that BoJ may target a higher inflation and lower nominal yield in the short run by relying more on negative IOER shock but this should be phased out soon because in the long run these two targets are not mutually consistent.

7 Conclusion

Hardly any county has ever experienced so many monetary policy rules and regime switches within a short period of time as Japan. On the other hand, fiscal policy has been relatively stable. In this paper, we set up a monetary business cycle model of the Japanese economy with a particular focus on the bond market. Using this model, we study the effect of various phases of quantitative easing on the bond market fluctuations in a macro financial setting. Quantitative easing is modelled as a positive shock to monetary base with an offsetting purchase of long term government bonds by BoJ which causes maturity transformation of commercial bank assets. Our study spans the period 1999:Q1 to 2017:Q1 over which the Japanese monetary policy underwent at least five transitions which include switch between interest rate control and monetary base control. We put both these control instruments in a new Keynesian model. We find that the monetary base fluctuations significantly explain macro financial fluctuations. Among the two policy rates namely overnight borrowing rate and IOER, our estimated DSGE model predicts that the latter explains aggregate fluctuations next to the QE shock.

In terms of the effects of unconventional monetary policy on bond market yields, we find that the traditional QE raises the nominal yield to maturity and the nominal holding period returns of all maturity bonds. This happens particularly because a QE shock triggers inflationary expectations which raise all nominal yields. The term premia of all maturity bonds decline in response to a positive QE shock while shorter maturity bonds experience larger drops. About policy rate changes, we find that a negative shock to IOER is more effective in stimulating the economy than lowering the overnight call rate. A negative IOER also lowers the nominal yield to maturity and raises inflation thus making positive inflation target with a zero yield achievable in the short run. The lesson for yield curve control experiment of BoJ is that a long term zero yield target is not consistent with a long run 2% inflation target. Thus it might be more effective to pursue the yield curve control only for the short run.

8 Appendix

8.1 Data Sources

The data for overnight call rate came from Bank of Japan. Regarding bond yield series, yield observations are based on the estimated yield curve by the Japanese Ministry of Finance. The amount of total bank reserve (required reserve and excess reserve) and the amount of monetary base came from BoJ sources. GDP data came from Economic and Social Research Institute, Cabinet Office. Employment Index is drawn from Monthly Labour Survey, Ministry of Health,Labour and Welfare. The series for CPI (all items) are from Statistics Bureau, Ministry of Internal Affairs and Communications. The proxy for the loan rate is the short term prime rate and is from Bank of Japan. Annualized real yields to maturity are computed as follows. Denote nominal yield to maturity of 10 year bond each quarter by $y^n_{t,10}$ (annual rate), real yield to maturity of 10 year bond each quarter by $y^r_{t,10}$, and the quarterly CPI price level by p^{CPI}_t. The quarterly inflation between t and t+1 is then calculated as $\pi_t = \frac{p^{CPI}_{t+1}}{p^{CPI}_t} - 1$. The annualized net CPI inflation for each quarter is calculated by $(1+\pi_t)^4 - 1$. Then annualized real yield to maturity is calculated from annual nominal yield to maturity and annualized inflation by $y^r_{t,10} = \frac{1+y^n_{t,10}}{(1+\pi_t)^4} - 1$.

The annualized net nominal holding period return for 10 year bond (denoted as $hpr^n_{t,10}$) is computed as follows. It is calculated from the nominal yield of 10 year bond and that of 9.75 year bond as follows: $hpr^n_{t,10} = [1 + \ln\{\frac{\exp(-9.75y^n_{t+1,9.75})}{\exp(-10y^n_{t,10})}\}]^4 - 1$. The annualized net real holding period return for 10 year bond is then: $rhpr^t_{t,10} = \frac{1+hpr^n_{t,10}}{(1+\pi_t)^4} - 1$. Term premium is the nominal yield to maturity of 10 year bond minus nominal overnight call rate. Regarding cyclical components of the level series (GDP, consumption, investment and employment), a Christiano-Fitzerald asymmetric band pass filter is used for computing cyclical components reported in Figure 3. For data input and model comparison of moments of the relevant series in Table 3, a one sided Hodrick-Prescott filter is used for comparability with model's demeaned series (See footnote 25).

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