Modelling Bond Yield : Case of Japan
Work in progress

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Abstract

Using the lens of a medium scale DSGE model, we analyze the macroeconomic effects of Japan's unconventional monetary policy which is known as Qualitative and Quantitative Easing (QQE). We model QE as a reserve injection by the Central Bank to the banking system. Our focus is on the Japanese bond market. We model heterogenous responses of the yield curves of coupon bonds of various maturities to a positive QE shock instead of the extant approach of modelling a single bond yield using an exponentially decaying coupon. Our model successfully replicates the negative effects of a positive QE shock on the nominal bond yields of various maturities which is on par with the experiment of yield curve control of Bank of Japan in recent years. Second, our model has the potential of replicating the observed nonlinear hump shape yield response of bonds with shorter maturity. The other features of our model are: (i) excess reserve demand function of commercial banks in response to liquidity risk, and (ii) linkage among central bank, commercial banks and the government via government bonds and bank reserve.

1 Introduction

During the last two decades, the Japanese economy experienced several episodes of monetary policy changes. Starting from an era of near zero interest rate, Bank of Japan (BoJ) officially implemented the Quantitative Easing (QE) policy to inject liquidity into the banking system from the beginning of the millennium. Following this, after seven years of experiment with a conventional monetary policy of interest rate targeting, BoJ adopted a Qualitative and Quantitative Easing (QQE). This policy features an explicit inflation target, GDP, short rate and long term bond yield targets which are the cornerstones of the unconventional monetary policy (UMP). From 1999 onward, Japan went through six monetary regimes as summarized in Table 1

<Table 1 comes here>

The central focus of our paper is on QE and QQE and its implications for the Japanese bond market. The correlation matrix in Table 2 documents that monetary expansion is associated with lower nominal bond yield during the QE and QQE period. The correlation coefficient between nominal bond yield and quarterly growth rate of monetary base is significantly negative. Second, the yield curve as seen in Figure 2 shows remarkable nonlinearity during the QE and QQE periods. The nonlinearity is manifested by the hump shaped yield curve of lower maturity bonds which yield negative returns during the most recent phase of QQE. Nonlinearity of yield curve is worthy of attention because normally we expect the yield curve to be monotonic rising or falling. BoJ first bought bond with relatively shorter maturities (initial QQE) to exploit a downward sloping term structure by hoping that the decline in yield of short maturity bonds will also lower the yields of long maturity. Since this did not happen, BoJ started to buy more aggressively both bonds of short maturity and 10 year long maturity (YCC) with specific target for yield levels (-0.1% for short rate and 0% for 10 year yield).

<Table 2 comes here>
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Understanding the yield curve is important for Japan. If the price of government bond falls and the yield rises, the Japanese government might not be able to refinance the current debt. At the end of December 2022. 48.1% of JGBs is held by BoJ and 14.4% is held by commercial banks and securities houses. Fluctuations of bond prices and yields impact commercial banks and the BoJ balance sheets. Modelling negative effect of monetary expansion on bond yields is a challenge because usually the Fisher's effect tends to dominate.

Our study is novel because we model heterogeneity of coupon bonds of different maturity. This task is important because the yield curves of bonds with different maturity show remarkable dissimilarity. For example as seen in Fig 1 shorter maturity bonds show significant nonlinearity compared to longer maturity bonds. In the extant literature, bond pricing is modelled by an exponentially decaying coupon value a la Woodford (1984). Such a formulation prices a consol with an exponentially decaying coupon and thus it is unable to reproduce the heterogeneity of yield curves. We explicitly price coupon varying maturities and derive the yield to maturity from the price equation.

Second, our formulation of QE intervention is different from the extant LSAP approach of Bernanke which involves an open market operation by the Central bank. We follow a traditional money multiplier approach as in Ryan and Whelan (2022), and Altavilla et al. (2021). The maturity transformation of banks' assets and resulting changes in yield happen as an equilibrium response to reserve injection of the CB.[1]

Third, our model is a stylized medium scale new Keynesian model similar to the extant models such as Smets and Wouters (2007), Gerali et al. (2010), Banerjee et al. (2019) and others. The advantage of using a DSGE model is

[1] Chistensen and Krogstrup (2018) report that a large increase in reserve at Swiss banks without changing the supply of long term government bonds triggered a decline in long term yields because of bank devitrification of assets.

that it enables us to see the linkage between the real and financial sectors of the economy when a policy shock hits the economy. The model has standard frictions such as aggregate habit persistence, investment adjustment cost, loan adjustment cost, monopolistic price formation, and nominal stickiness. There are two nonstandard features of our DSGE model. First, we model banking friction resulting from a liquidity risk similar to Chang et al. (2014) and Banerjee et al. (2019). A liquidity risk in the form of an anticipated negative cash flow shock disciplines commercial banks to hold precautionary excess reserve and not to push loans recklessly. In our setting, banks solve a dynamic cash flow asset management problem taking deposits as given. A QE intervention means that banks are supplied with more reserve and then banks optimally reallocate its portfolio between reserve, loans and government bonds. Such asset reallocation impacts the bond yields and loan interest rates. Banks in our model hold all the government bonds keeping with the Japanese institutional reality.

Finally, we explicitly study the dynamic link among the government, central bank (CB hereafter) and commercial banks via the long term government bonds and bank reserves by formulating the CB's resource constraint in line with the recent work of Hall and Reis (2015). This helps us study the general equilibrium consequences of QQE policy for the bond market when the CB creates reserve. The government in our model plays a passive role in spending an exogenous stream of final goods and finances it from lump sum tax as well as consumption tax on households.[2]

In our model, the key transmission channel of monetary policy effect on the bond market yields is consumption smoothing. This consumption smoothing manifests by a time varying pricing kernel which has an external habit persistence component. The hump shape of the yield curve arises from the hump shaped response of consumption/GDP ratio in our model. The production sector imposes restrictions on the consumption space which gives rise to this hump shaped response of consumption ratio to monetary shock. Consumption ratio rises in response to a positive monetary shock because QE makes banks demand more bonds and advance less loans. Since investment is fully intermediated, less loans discourage investment and stimulates consumption in the short run but then inflation tax resulting from QE lowers consumption.

The paper is organized as follows. In the next section, we briefly review the extant literature on the modelling of the Japanese economy. In section 3, our basic DSGE model is laid out. Section 4 is devoted to present quantitative analysis of the model. Section 5 concludes.

2 Connections to literature

There is a growing literature on DSGE modelling of the Japanese economy. Sugo and Ueda (2007) is one of the first articles that estimate a DSGE model of the Japanese economy. Although they model monetary policy rule and use call rate

[2] As for the fiscal year 2022, the consumption tax is around 34.3% of tax revenue for the government. Thus it constitutes an important part of tax income.

as a proxy for the short term nominal interest rate, they do not explicitly model the role of CB and abstract from any analysis of monetary or fiscal policy effects on bond market except that there is an interest rate shock through a discount bond. Iwata (2009) focuses on the fiscal policy under DSGE setting. Hirose (2020) estimates a DSGE model with a deflationary steady state for Japan and considers whether several shocks to the economy have an inflationary effect. McNelis and Yoshino (2016) compare the performance of three policy rules on reducing the government debt using a DSGE model. However, they do not explicitly model the role of CB and there is no government bond in the model. Fueki et al. (2016) set up a DSGE model to analyze potential output and output gap for the Japanese economy.

There is a growing volume of empirical literature on QE effects. Adjemian and Juillard (2010) estimate a DSGE model with a zero lower bound for nominal interest rate. Michaelis and Watzka (2017) consider the change in the effectiveness of quantitative easing policy at the zero lower bound. Although there are liquidity shocks in their model, they do not have a DSGE model. Instead they estimate time varying parameter using VAR analysis and do not study the effect of monetary policy on bonds. Hayashi and Koeda (2019) consider the effect of QE on macroeconomy under the framework of regime-switching structural VAR. They show that a higher reserve raises inflation and output when the nominal policy rate is close to the effective lower bound of interest rate on excess reserve. Nagao et al. (2021) measures the magnitude of both conventional and unconventional monetary policy shocks from the term structure of interest rate and show that the magnitude of monetary policy shocks on the macroeconomic variables are modest in a VAR setting when both the short and long term interest rates are close to the lower bound of zero. Koeda (2019) estimates a 5 variable structural VAR with an effective zero lower bound and showed that QQE increased output.

There is a third strand of literature which focuses on portfolio behavior of Japanese banks involving loan and reserve. Ogura (2020) models the static profit maximizing behavior of regional banks and regional loan demand function by individuals. He shows empirically that increase in liquidity ratio of banks caused by quantitative easing led to more competition among banks. Shioji (2019) investigates the effect of QE on bank loan using panel data of bank balance sheets. He finds that banks did extend loans but the estimated effect of on loan/asset ratio is very close to zero. Shioji (2020) also estimates the effect of QE on bank based on the panel data of regional banks and finds that the result depends on the sample period. However even in the sample period where banks extended loan due to QE, the effect of QE on bank loan was not large.

In the backdrop of these extant studies, our study has two novel features. First, we explicitly model the role of CB and the nexus between the government budget constraint, the CB budget constraint and the commercial bank's budget constraint in a DSGE framework. We focus on the transmission channels of BoJ's QQE policy to the Japanese bond market by exploiting the dynamic linkage among CB, commercial banks and the government through bond holdings and the bank reserve as in Hall and Reis (2015). Second, we analyze the effect of negative IOER on the aggregate economy.[3] Such an exercise is also internationally relevant because before BoJ introduced IOER in 2016, ECB had already introduced on June 5, 2014 a negative rate on excess bank reserve of the Eurosystem.[4]

While a plethora of literature exists on various applications of DSGE models, what is less understood is its bond yield implications. Rudebusch and Swanson (2012) show some innovative applications of a DSGE model to understand bond pricing behavior. However, they do not focus on the monetary policy effects on the bond market behavior, which is the scope of our study. Chen et al. (2012) is one of the few studies that uses DSGE modelling to assess the effects of UMP on long term bond yields in the US who find that the QE in the US has rather insignificant effects on long term bond yield. They, however, do not formulate the CB balance sheet and commercial banks' asset portfolio which we do. Moreover, their focus is on the QE operation in the US, while we focus on QQE in Japan which involves additional monetary policy instruments including IOER.

Our paper is closest to Sudo and Tanaka (2021) who investigate the effect of QE on long term and short term yields in a DSGE model with segmented bond market. However, they do not explicitly model dynamic portfolio behavior of banks involving the choice of precautionary excess reserve, loans and government bonds with portfolio adjustment cost. Our monetary policy transmission channel, therefore, works via the change in the asset mix of banks and its interaction with the portfolio adjustment cost. In Sudo and Tanaka (2021), the QE effects work via the differential responses of households in a segmented bond market where a group of households do not hold any short term government bonds. We model the bond market segmentation by assuming that only banks hold long term government bonds and deal with BoJ when QE operation takes place. This feature is also consistent with the data (see Fig 2 later). The monetary transmission channel in our setting is thus fundamentally different from Sudo and Tanaka (2021).

3 Model

The building blocks of the model are similar to Banerjee et al. (2019). We have seven players in the economy: the representative household, three types of firms, commercial banks, CB and the government. Household owns all productive units. Household's saving consists of short term bank deposits which provide convenience utility and one period discount bonds issued by the government. They supply labour to wholesale goods firms. Their income consists of labour, interest income from deposit and yields from discount bonds, cash flows generated from the ownership of firms and the commercial banks.

[3] Koeda (2019) investigates the effect of exit from IOER intervention in a structural VAR setting while we explore the effect of IOER drop on the aggregate economy within DSGE setting.

[4] https://www.ecb.europa.eu/press/pr/date/2014/html/pr140605_3.en.html

There are three types of firms: wholesale, capital goods and retail firms. Competitive risk neutral one period lived wholesale firms finance their capital spending from banks. Competitive capital goods firms buy used capital from wholesalers and refurbish it to new capital using investment goods bought from retail firms. Retail firms costlessly convert wholesale goods to differentiated final goods and have some monopoly power of price fixing. Final goods can be used for household consumption, capital goods producers' investment and government use.

Retails banks collect household deposit to intermediate this to wholesale firms and also hold long term government bonds of varying maturities. Banks hold excess reserves since they anticipate an aggregate liquidity risk in the form of negative cash flow shock. If the size of this liquidity shock exceeds the current bank reserve, banks borrow from the lender of the last resort, CB at a penalty rate. Banks are exposed to loan default risk because a fraction of loans become non-performing.

The government consumes some final goods financed by lump sum taxes and consumption taxes on households and borrowing from the commercial banks and the CB via issuing long term government bonds. The CB finances its government bond holding by reserve creation, seigniorage and the revenue earned from banks resulting from penalty loans.

3.1 Households

The representative household solves the following maximization problem:

$$\max_{\{c_{t+j}, D_{t+j}, M_{t+j}^{TD}, H_{t+j}\}} E_t \sum_{j=0}^{\infty} \beta^j [U(c_{t+j} - \gamma_c C_{t+j-1}) + V(D_{t+j}/P_{t+j}) + W(M_{t+j}^{TD}/P_{t+j}) - \Phi(H_{t+j})]$$

subject to the flow budget constraint:

$$P_t c_t \ + D_t + M_t^{TD} \leq W_t H_t + (1 + i_t^D) D_{t-1} + M_{t-1}^{TD} + TR_t \quad (1)$$

where c_t is the representative agent's consumption basket, C_{t-1}= aggregate consumption in the previous period, γ_c = external habit persistence parameter [5]., D_t is one period nominal deposit, P_t is aggregate price index, M_t^{TD} is nominal transaction demand for cash and H_t is labour hours. W_t is nominal wage, i_t^D is an exogenous risk-free nominal interest rate on deposits and TR_t is the nominal lump sum transfer to the household which includes cash flows from capital goods firms, retail goods firms, commercial banks as well as transfer from the government. We assume that household receives direct utility from bank deposits and cash holding.[6] $U(.), V(.), W(.)$ are instantaneous continuous, strictly

[5] As in Gali (2015), we assume that the household receives unity from a CES consumption aggregator of continuum of differentiated goods over a unit interval with the elasticity of substitution equal to ε^Y which characterizes the price elasticity of demand of the i^{th} differentiated good.

[6] We put both real cash balance and real deposits in the utility function motivated by the fact that both money and short term bank deposits provide different kinds of transaction

concave utility functions from consumption, real deposit and real money balance with the usual regularity conditions and $\Phi(H_t)$ is the continuous disutility function from work.

The first order conditions are:

$$D_t: \; U_{ct} = V'(d_t) + \beta E_t U_{ct+1}(1 + i^D_{t+1})/(1 + \pi_{t+1}) \tag{2}$$

$$M^{TD}_t: \; U_{ct} = W'(m^{TD}_t) + \beta E_t U_{ct+1}/(1 + \pi_{t+1}) \tag{3}$$

$$H_t: \Phi'(H_t) = (W_t/P_t)\; U_{ct} \tag{4}$$

where U_{ct} is the derivative of $U(c_t - \gamma_c C_{t-1})$ with respect to c_t, $d_t = D_t/P_t$ is the real deposit, $\pi_{t+1} = (P_{t+1}/P_t - 1)$ is the net inflation rate and $m^{TD}_t = M^{TD}_t/P_t$ is the real transaction demand for cash. Equation (2) shows that marginal utility cost of holding a dollar of deposit balances the temporal marginal utility of liquidity service from deposits and the discounted utility benefits of the interest on deposit adjusted for inflation tax. Likewise equation (3) shows the marginal equivalence condition of cost and benefit of holding a dollar money balance. Equation (4) is the standard static efficiency condition for labour supply.

3.2 Production Sector

3.2.1 Capital goods producing firms

Capital goods producers buy last period's used capital $\{(1-\delta_k)\,K_{t-1}\}$ from the wholesale firms/entrepreneurs at a real price Q_t where $K_{t-1}$is previous period's capital and δ_k is the physical rate of depreciation of capital. They produce new capital stock K_t by investing I_t of final goods using a linear investment technology:

$$K_t = (1-\delta_k)K_{t-1} + Z_{xt} I_t \tag{5}$$

where Z_{xt} is an investment specific technology shock which evolves as follows:

$$Z_{xt} = \overline{Z_x}^{1-\rho_z} Z^{\rho_z}_{xt-1}\; exp(\xi^z_t)$$

where $\overline{Z_x}$ is its the steady state level, ρ_z is the serial correlation coefficient and ξ^z_t is a stationary noise to be specified later. After investment, this new capital is sold to the wholesalers at a real price Q_t. For one unit investment, the capital goods producers purchase $[1 + \Xi\left(\frac{I_t}{I_{t-1}}\right)]$ of final goods where $\Xi(.)$ is

convenience to the household. Putting real cash balance in the utility function has a long tradition following Sidrauski (1967). The idea of real deposits in the utility function is borrowed from Hansen and Imrohoroglu (2016) who put short term government bonds in the utility function. Since households value the liquidity service of short term bank deposits, they are willing to accept a lower rate on bank deposits than the loan rate the banks charge to the wholesale goods firms which are also owned by households. A natural borrowing-lending spread or limits to arbitrage thus arises in our model (see footnote 22).

a convex flow investment adjustment cost function with $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) = \kappa$.[7] The capital goods producer then solves

$$\max_{\{I_{t+j}\}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} CF^K_{t+j}$$

where $\Omega_{t,t+j}$ is the inflation adjusted stochastic discount factor[8] between t and $t+j$ which is equal to $\frac{\beta^j U_{ct+j}}{U_{ct}} \cdot \frac{1}{1+\pi_{t+j}}$ and CF^K_t is the cash flow of the capital goods producer given by:

$$CF^K_t = P_t \left[Q_t I_t - \left\{ 1 + \Xi\left(\frac{I_t}{I_{t-1}}\right) \right\} I_t \right]$$

The first order condition gives the following Euler equation similar to Gertler and Karadi (2013):

$$Q_t = 1 + \Xi\left(\frac{I_t}{I_{t-1}}\right) + \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - E_t \frac{\beta U_{ct+1}}{U_{ct}} \left[\Xi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right] \quad (6)$$

3.2.2 Wholesale goods producing firms

There are continuum of risk neutral wholesale firms over the unit interval. The i^{th} wholesale firm produces intermediate goods ($Y^W_t(i)$) for the i^{th} final goods producing retailer. For doing so, it hires labour from the households and purchases new capital from the capital good producing firms. This firm takes a loan of $L_t(i)$ from the bank in order to cover the nominal cost of new capital, $P_t Q_t K_t(i)$ where Q_t is the real price of capital. We assume that all capital spending is debt financed. Used capital at date t is sold at the resale market at the price Q_t.

Balance sheet condition of the typical wholesale firm is:

$$L_t(i) = P_t Q_t K_t(i) \quad (7)$$

The wholesale goods production function is specified as follows:

$$Y^W_t(i) = A_t K^{\alpha}_{t-1}(i)(\Theta_t H_t(i))^{1-\alpha} \quad (8)$$

where A_t is the TFP shock , $0 < \alpha < 1$, and Θ_t is a labour augmenting technical progress component. The TFP shock evolves as follows:

$$A_t = \overline{A}^{1-\rho_A} A^{\rho_A}_{t-1}\, exp(\xi^A_t)$$

[7] We assume a quadratic investment adjustment cost function: where $\Xi\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left[\frac{I_t}{I_{t-1}} - 1\right]^2$. Note that this investment adjustment cost is incurred before investment is undertaken to install new capital K_t. That is why it does not appear in the linear investment technology (5).

[8] Since the household owns all firms and banks, **these firms** and banks also share the same stochastic discount factor.

where $\overline{A}$ is its steady state level, ρ_A is a serial correlation coefficient and ξ_t^A is a stationary noise to be specified later. We assume that Θ_t grows at a deterministic gross rate Λ which is the balanced growth rate of the economy.

The equilibrium real wage is $W_t/P_t = (1-\alpha)\frac{(P_t^W/P_t)Y_t^W(i)}{H_t(i)}$, where P_t^W is the nominal price of the wholesale good.

The gross real rate of return from capital is given by,

$$1+r_{t+1}^k(i) = \frac{(P_{t+1}^W/P_{t+1})Y_{t+1}^W(i) - (W_{t+1}/P_{t+1})H_{t+1}(i) + (1-\delta_k)\,Q_{t+1}K_t(i)}{Q_tK_t(i)}$$
$$= \frac{(P_{t+1}^W/P_{t+1})\left(\frac{Y_{t+1}^W(i)}{K_t}\right) - (1-\alpha)\frac{(P_{t+1}^W/P_{t+1})Y_{t+1}^W(i)}{H_{t+1}(i)}\left(\frac{H_{t+1}(i)}{K_t(i)}\right) + (1-\delta_k)\,Q_{t+1}}{Q_t}$$
$$= \frac{(P_{t+1}^W/P_{t+1})MPK_{t+1}(i) + (1-\delta_k)Q_{t+1}}{Q_t}$$

where $MPK_{t+1}(i)$ denotes the i^{th} firm's marginal product of capital at date $t+1$. Defining i_{t+1}^L as the net nominal interest rate on loans betweent and $t+1$, the optimality condition for firms' demand for capital (or the no arbitrage condition) can be written as

$$1+r_{t+1}^k(i) = \left(1+i_{t+1}^L\right)/(1+\pi_{t+1})$$

which yields,

$$1+i_{t+1}^L = \frac{P_{t+1}^W MPK_{t+1}(i) + (1-\delta_k)\,P_{t+1}Q_{t+1}}{P_tQ_t}$$

In other words,

$$1+i_{t+1}^L = \left[\left(\frac{P_{t+1}^W}{P_{t+1}}\right)\frac{MPK_{t+1}(i)}{Q_{t+1}} + 1 - \delta_k\right]\left[\frac{(1+\pi_{t+1})Q_{t+1}}{Q_t}\right] \tag{9}$$

Since all firms face the same loan rate, i_t^L and capital price, Q_t, they all produce the same output in equilibrium.

3.2.3 Retail firms

Similar to Bernanke et al. (1999), there are continuum of retails firms over the interval. The i^{th} retailer buys intermediate goods at price P_t^W and package them into final goods and operate in a monopolistically competitive environment. The i^{th} retailer convert the i^{th} variety of the intermediate goods, $Y_t^W(i)$, one-to-one into differentiated final good, $Y_t(i)$ at zero cost. Each retailer sells his unique variety of final product after applying a markup over the wholesale price, and factoring in the market demand condition which is characterized by price elasticities $\left(\varepsilon^Y\right)$.[9] Retailer's prices are sticky and indexed to past and steady

[9] As in Rotemberg (1982), each retail firm continuously adjusts its price subject to a quadratic price adjustment cost and maximizes the present value of cash flows subject to differentiated final demand function. We omit the details of the decision problem of the retail firms, which are quite standard. See Basu and Sarkar (2016) for details of the retailer's problem.

state inflation as in Gerali et al. (2010) and Banerjee et al. (2019) based on the indexation parameter $\theta_p \in (0,1)$. Retailers bear a quadratic adjustment cost given by ϕ_p if they want to change their price over and above what indexation allows.[10]

The first order condition after imposing a symmetric equilibrium is standard:

$$1-\varepsilon^Y+\varepsilon^Y(P_t/P_t^W)^{-1}-\phi_p\left\{1+\pi_t-(1+\pi_{t-1})^{\theta_p}(1+\overline{\pi})^{1-\theta_p}\right\}$$
$$+E_t\Omega_{t,t+1}\phi_p\left\{1+\pi_{t+1}-(1+\pi_t)^{\theta_p}(1+\overline{\pi})^{1-\theta_p}\right\}(1+\pi_{t+1})^2\frac{y_{t+1}}{y_t}=0 \quad (10)$$

In the steady state, when $\pi_{t+1}=\pi_t=\pi$, the above price equation reduces to a simple static markup equation:

$$\frac{P_t}{P_t^W}=\frac{\varepsilon^Y}{\varepsilon^Y-1}. \quad (11)$$

3.3 Banks

As in Banerjee et al. (2019) the banking problem is nonstandard in our setting. Commercial banks solve a dynamic portfolio choice problem involving three assets, namely reserve holding, N types of government coupon bonds (JGB) with varying maturities and loans. Denote outstanding nominal loans issued at date $t-1$ as L_{t-1}, i_t^L is the loan interest rate between $t-1$ and t Let j stand for the maturity at issue of the j^{th} bond in bank's portfolio. Denote the outstanding number of government bonds held by the commercial banks at date t as $b_t^{P,j}$ and P_{ct}^j be its nominal price where $j=2,3,...,N$ assuming that the lowest maturity period is 2. Denote M_t^{RD} as commercial banks' outstanding reserve holding at date t.

Banks are subject to a statutory reserve requirement as follows:

$$M_t^{RD}=\alpha_r D_t \text{ for all } t \quad (12)$$

where α_r is the legal reserve ratio.

Bank's cash flow at date t can be rewritten as:

$$CF_t^B=\sum_{j=2}^{N}(C+P_{ct}^{j-1})b_{t-1}^{P,j-1}-\sum_{j=2}^{N}P_{ct}^j b_t^{P,j} \quad (13)$$

$$-(L_t-(1+i_t^L)L_{t-1})-0.5\varpi(L_t-L)^2+(1+\overline{i}^R)M_{t-1}^R-(1+i_t^p)\chi_t(\zeta_t-M_{t-1}^{RD})$$
$$-(1-\chi_t)\zeta_t-M_t^{RD}+(D_t-(1+i_t^D)D_{t-1}) \quad (14)$$

where χ_t, is an indicator function that takes the value unity if $\zeta_t-M_{t-1}^{RD}>0$ and zero otherwise and ϖ is a loan adjustment cost parameter. Given the deposit

[10] As in any standard new Keynesian model, the nominal rigidity is quite crucial for generating real effects of the monetary policy.

sequence $\{D_t\}$, banks choose $\{M_t^{RD}, b_t^{P,j}, L_t\}$ which solve the following dynamic optimization:

$$\max_{\{M_{t+j}^{RD}, b_{t+j}^{P,j}, L_{t+j}\}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} CF_{t+j}^{B}$$

s.t. the statutory reserve requirement (12).

The Euler equations for bonds ($b_t^{P,j}$) and loans (l_t) in real terms (denoted as lower cases) are written as follows:

$$b_t^{P,j} : P_{ct}^{j} = E_t(C + P_{ct+1}^{j-1})\Omega_{t,t+1} \tag{15}$$

$$L_t : \ L_t = \bar{L} + \frac{E_t\Omega_{t,t+1}(1+i_{t+1}^{L}) - 1}{\varpi} \tag{16}$$

$$M_t^{RD} : \frac{M_t^{RD}}{D_t} = 1 - \frac{1 - (1+i_{t+1}^{R})E_t\Omega_{t,t+1}}{(1+i_{t+1}^{p})E_t\Omega_{t,t+1}} \tag{17}$$

Bank deposit rate, i_t^D is regulated by the government and we set it close to zero as seen in the Japanese economy during the last two decades.[11]

3.4 The Central Bank budget constant

CB's real flow budget constraint is as follows:

$$\begin{aligned} m_t^R &= (1+i^R)/(1+\pi_t)m_{t-1}^R \\ &+ \sum_{j=2}^{N} P_{ct}^{j} b_t^{CB,j} - \{m_t^T - m_{t-1}^T/(1+\pi_t)\} \\ &- \sum_{j=2}^{N} (C + P_{ct-1}^{j-1})/(1+\pi_t) b_{t-1}^{CB,j-1} + div_t \end{aligned} \tag{18}$$

where m_t^R is the real reserve, m_t^T is the supply of real cash balance, $b_t^{j,CB}$ is the holding of government bond by CB, div_t = dividend paid to the government as in Hall and Reis (2015). Literally, the BoJ does not pay such dividend but it should generate sufficient revenue to cover the deficits of the government. Thus the dividend is the link between BoJ and the government.

[11] Note that there is a borrowing-lending spread because deposit appears in the utility function and provides transaction convenience to the household. To see it combine (2) and (16) to get the following steady state borrowing-lending spread:

$$i^L - i^D = \frac{(1+\pi)}{\beta}\frac{V'(d)}{U_c(c)} > 0$$

3.5 Government budget constraint

The real government budget constant is:

$$G_t + \sum_{j=2}^{N}(C + P_{ct-1}^{j-1})\frac{b_{t-1}^{G,j-1}}{1+\pi_t} = T_t + \sum_{j=2}^{N} P_{ct}^{j} b_t^{G,j} + div_t \tag{19}$$

where G_t is real government consumption and $b_t^{G,j}$is the supply of government bond at t.

The real government spending (G_t) has the following exogenous law of motion:

$$G_t = \Lambda^t \ \widetilde{G}_t$$

where Λ is the balanced growth rate as specified in (8) and the stationarized government spending shock ($\widetilde{G}_t$) follows the process:[12]

$$\widetilde{G}_t = \overline{G}^{1-\rho_G} \widetilde{G}_{t-1}^{\rho_G} \exp(\xi_t^G) \tag{20}$$

3.6 Equilibrium

1. Goods market clears: $y_t = c_t + i_t + G_t$ + *investment and price adjustment costs.*

2. The loan market clears:

$$L_t/P_t = Q_t K_t \tag{21}$$

3. The bond market clears:

$$b_t^{P,j} + b_t^{CB,j} = b_t^{G,j} \text{ for all } j \tag{22}$$

4. Money market clears which means that the demand for bank reserve (M_t^{RD}) equals the supply of bank reserve (M_t^R) and the transaction demand for money (M_t^{TD}) equals the corresponding supply (M_t^T):

$$m_t^{RD} = m_t^R \tag{23}$$

$$m_t^{TD} = m_t^T \tag{24}$$

3.7 Integrated government budget constraint

Plugging (18) into (19), we get the integrated government budget constraint as follows:

$$G_t - T_t = \sum_{j=2}^{N} P_{ct}^{j} b_t^{P,j} - \sum_{j=2}^{N}(C + P_{ct-1}^{j})\frac{b_{t-1}^{P,j}}{1+\pi_t} \tag{25}$$
$$+ m_t^R - \frac{(1+i_t^R)}{(1+\pi_t)} m_{t-1}^R + m_t^T - \frac{m_{t-1}^T}{(1+\pi_t)}$$

[12] The stationarized level variables are written with $\widetilde{}$.

which basically state the government deficit is financed issuing bonds to the private sector which comprises commercial banks in our setting and seigniorage revenue from inside and outside money.

3.8 Government debt and bond supply block

We now turn to the composition of government debt. The outstanding real government debt (d_t) by treating P_t as the numeraire is given by

$$d_t = \sum_{j=1}^{N} P_{ct}^{j} b_t^{G,j} \tag{26}$$

where $b_t^{G,j} = b_t^{P,j} + b_t^{CB,j}$.

We assume that $\varkappa^j$ fraction of total number shares issued by the government is held by the CB which means $b_t^{P,j} = (1-\varkappa^j) b_t^{G,j}$ and $b_t^{CB,j} = \varkappa^j b_t^{G,j}$.

Given this allocation of each bond between CB and the commercial banks, the debt held by the commercial banks (d_t^p) (which we call public debt) is given by

$$d_t^p = \sum_{j=1}^{N} P_{ct}^{j} (1-\varkappa^j) b_t^{G,j} \tag{27}$$

Notice that the public debt depends on the allocation of various bonds between the CB and the banks which is summed up by κ^j. For calibration purpose, we assume that κ^j is the same for all j which means

$$d_t^p = (1-\varkappa) \sum_{j=1}^{N} P_{ct}^{j} b_t^{G,j} \tag{28}$$

We assume that the total government debt (26) a constant fraction (η) of the real market value of capital. In other words,

$$d_t = \eta Q_t K_t \tag{29}$$

Plugging (28) into (29) and defining $\eta(1-\varkappa)$, we get

$$d_t = \lambda Q_t K_t \tag{30}$$

3.9 Monetary Policy

QE means a reserve injection into the banking system with a long run inflation target $\overline{\pi}$ as follows:

$$\frac{1+\mu_t}{1+\overline{\pi}} = \left(\frac{1+\mu_{t-1}}{1+\overline{\pi}} \right)^{\rho_\mu} \exp(\xi_t^\mu) \tag{31}$$

where $1+\mu_t = M_t^R/M_{t-1}^R$, $\rho_\mu \in (0,1)$ and ξ_t^μ is a QE shock with a forcing process to be specified later. Such a money supply process imposes restriction on the short run growth rate of real reserve and inflation as follows:

$$\frac{(1+\pi_t)(m_t^R/m_{t-1}^R)}{1+\overline{\pi}} = \left(\frac{(1+\pi_{t-1})(m_{t-1}^R/m_{t-2}^R)}{1+\overline{\pi}}\right)^{\rho_\mu} \exp(\xi_t^\mu) \tag{32}$$

4 Shocks

4.1 Forcing processes

We assume (note:do we ignore pr shocks which exist in the code? I believe it does not affect the result of the paper as we)(the following specifications for the TFP shock ξ_t^A, IST shock ξ_t^z, government spending shock ξ_t^G, QE shock ξ_t^μ:

$$\xi_t^j = \theta_j \xi_{t-1}^j + \epsilon_t^j, \text{ for } j = A, z, \widetilde{G}, \mu \tag{33}$$

where $\{\epsilon_t^j\}$ is an *i.i.d.* process for all $j = A, z, G, \mu$.

5 Calibration

The quarterly deposit rate is the sample average of the quarterly ordinary deposit rate. The quarterly call rate is the sample average of quarterly overnight call rate. The quarterly reserve interest rate was set at zero. Quarterly GDP growth rate is based on quarterly real per capita closed economy GDP growth rate (closed economy GDP is equal to private consumption+private nonresidential investment+government consumption). The habit persistence parameter κ is fixed at 0.67 based on Basu and Wada (2022). The SS government consumption is set to match key ratios. We set the quarterly steady state inflation at zero as the sample average is 0.000.

The SS TFP parameter is set to match key ratios. The SS investment specific shock is normalized at one. We set the quarterly time preference, quarterly capital depreciation and capital share at 0.99,0.009 and 0.314 which are in line with the values in Sugo and Ueda(2007). Price markup ratio and bond/loan ratios are chosen to mach key ratios. The quadratic adjustment cost is from Iwasaki et al. (2022) and we set the loan adjustment cost at a modest level of 4.

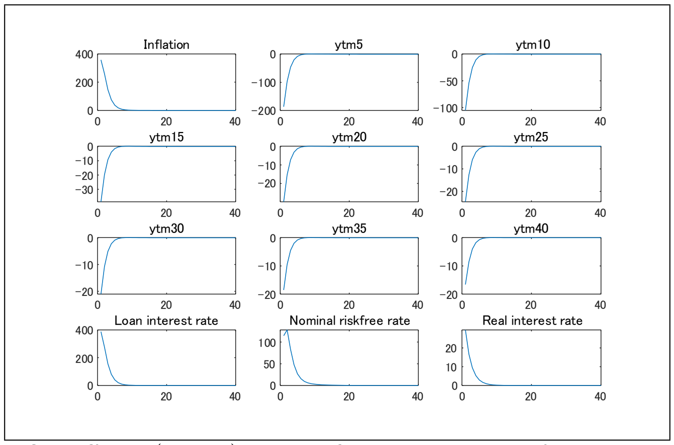
Parameter	Description	Value
$\overline{i^D}$	quarterly deposit rate	0.0000947
$\overline{i^p}$	quarterly call rate	0.000126
$\overline{i^R}$	quarterly reserve interest rate	0.00
Λ	quarterly GDP growth rate	1.0013
γ_c	habit persistence	0.67
$\overline{G}$	SS government consumption	1.85
$\overline{\pi}$	quarterly steady sate inflation	0

Parameter	Description	Value
$\overline{A}$	SS TFP	1.01
$\overline{Z_x}$	SS investment specific shock	1
β	quarterly time preference	0.99
δ_k	quarterly capital depreciation	0.009
α	capital share	0.314
ε^Y	price markup ratio	6
$\overline{\lambda}$	long run bond/loan ratio	0.00205
ϕ_p	quadratic price adjustment cost	30.8
ϖ	Loan adjustment cost	4

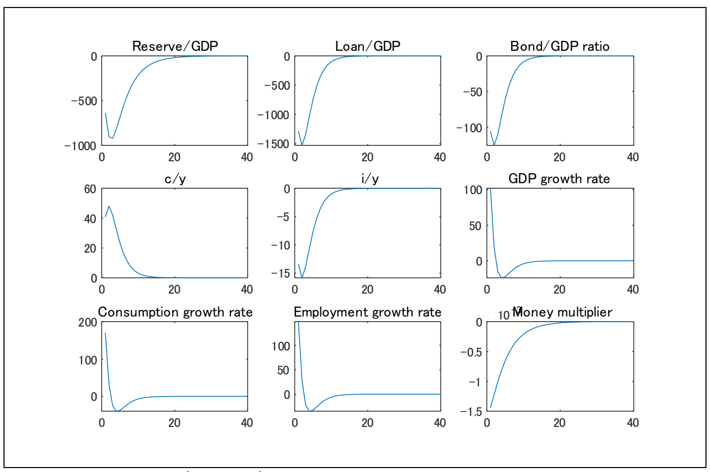
Parameter	Description	Value
θ_A	TFP	0.4671
θ_Z	Investment	0.5126
θ_μ	QE	0.4730
θ_G	Government	0.4468
θ_P	Price	0.20925
$std(\epsilon_t^A)$	size of a shock	0.01
$std(\epsilon_t^Z)$	size of a shock	0.01
$std(\epsilon_t^\mu)$	size of a shock	0.01
$std(\epsilon_t^G)$	size of a shock	0.01
$std(\epsilon_t^P)$	size of a shock	0.01
$std(\epsilon_t^\zeta)$	size of a shock	0.01

Parameter	Description	Value
ρ_A	TFP	0.8170
ρ_Z	Investment	0.9434
ρ_μ	QE	0.7935
ρ_G	Government	0.9007
ρ_ζ	etha	0.9

Parameter	observed value	calibrated value
consumption/GDP	0.624	0.600
investment/GDP	0.167	0.132
government consumption/GDP	0.208	0.268
domestic bank bond holding/GDP	1.05	1.05



QE effects (in BP) on the financial markets for $\gamma = 2$.



QE effects (in BP) on aggregat ecoonomy for $\gamma = 2$

5.0.1 Intuitions

Why do nominal yields go down while real interest rate and nominal loan rate rise in response to QE shock? The bond price equation involves sequence of inflation adjusted stochastic discount factors, $m_{t,t+j}$. Note that in our model with habit

$$m_{t,t+j} = \frac{\beta^j (c_t/c_{t-1})^{-\gamma} (c_{t+j}/c_t)^{-\gamma} (c_t/c_{t-1} - 1)^{-\gamma}}{1 + \pi_{t+j}}$$

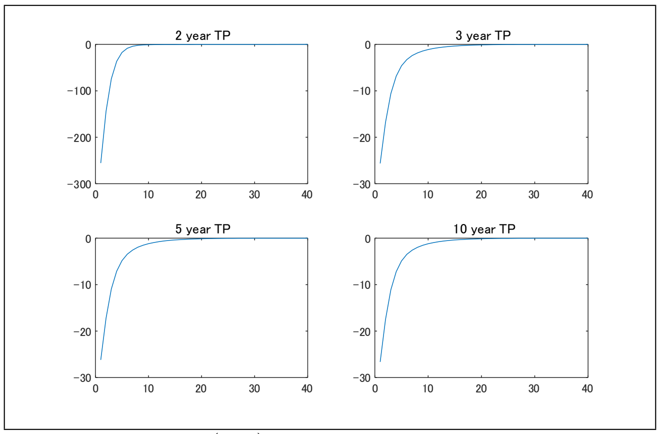
In response to QE shock, the cumulative inflation rates,π_{t+j} rise. Cumulative growth rates of consumption, c_{t+j}/c_t fall. Rise in the current consumption growth rate (c_t/c_{t-1}) explains why the real interest rate rises. However, fall in the cumulative growth rates of consumption and rise in cumulative inflation rates together explain the rise in bond price which means nominal yields fall.

The nominal loan rate rises because both the real interest rate and inflation rise.

5.0.2 Term Premia

The term premium is the difference between the one period holding return minus the risk free rate:

$$TP_t^j = E_t\left[\frac{P_{ct+1}^{j-1}+C}{P_{ct}^j}\right] - RF_t \text{ for } j = 1..N.$$



QE effects (BP) on Term Premia for $\gamma = 2$

5.0.3 QE effects on the term structure of interest rates

When we set the risk aversion coefficient at 0.5, we see a hump shaped response of yields from SS with respect to QE shock which lasts about 6 quarters. The immediate response is largest and positive for bond with maturity of 2Q at 73BP and the response keeps decreasing and becomes negative for bond with 5Q maturity, reaching the lowest level of -56BP for the bond with 11Q maturity. Then the response starts to rise and reaches -18BP for bond with 40Q maturity. Since the SS value of all the yields are 114BP, the all the levels after QE shock is positive.

When we set the risk aversion coefficient at 2, we see a uniform response of yields from SS with respect to QE shock which lasts about 5 quarters.

If we lower the value of price adjustment cost from the conventional value of 30.8 to 2.5, we can match the correlation between monetary base growth rate and yields. If we raise the value of QE shock from 0.4730 to 0.85, we can match the correlation between monetary base growth rate and yields. Compared with the case for risk aversion coefficient at 0.5, the immediate response is much larger in absolute value. The decline is largest for bond with 2Q maturity at -245BP and the absolute value of decline keeps getting smaller and it is -17BP for bond with 40Q maturity. Since the SS yields are 114BP, the yield level after

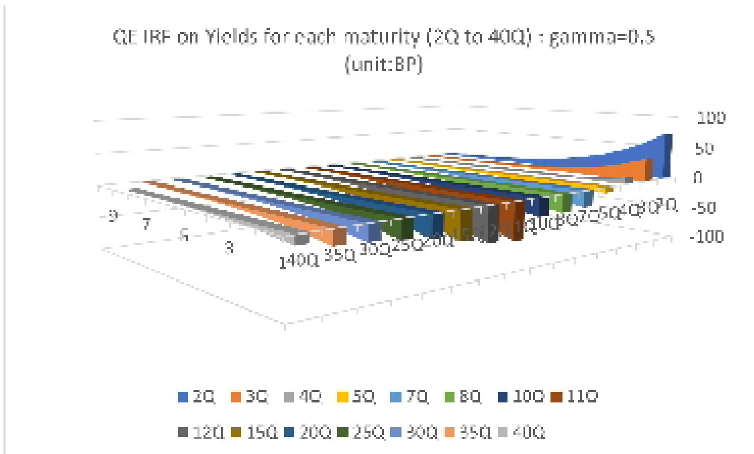


Figure 1: QE effects on Yields ($\gamma = 0.5$,unit:BP)

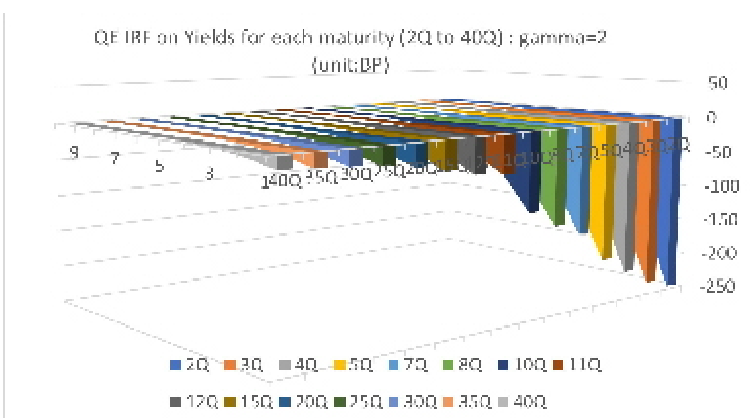


Figure 2: QE effects on Yields ($\gamma = 2$,unit:BP)

shock is negative for bonds with maturities from 2Q to 8Q. The level of yields after shock is positive for bond with maturities from 10Q to 40Q.

Seasonally unadjusted monetary base growth with bond yields

$\gamma = 2$, $\phi_p = 2.5$

2 year yield	3 year yield	5 year yield	10 year yield
-0.31	-0.30	-0.32	-0.32

Seasonally unadjusted monetary base growth with bond yields

$\gamma = 2$, $\theta_\mu = 0.85$

2 year yield	3 year yield	5 year yield	10 year yield
-0.27	-0.24	-0.28	-0.31

6 Conclusion

The goal of this paper is to assess the effect of QQE on bond yield in intertemporal general equilibrium using Japan as the testbed. We model a set of coupon bonds in equilibrium as opposed to a single coupon bond with infinite maturity. This is because we cannot assess the effect of monetary policy or any policy on the term structure of interest rates. We find that an expansionary monetary policy mostly depresses bond yield as well as term premia for a moderate level of risk aversion coefficient and our model has the potential to explain the hump shaped response of monetary policy on the yield curve for a lower value of risk aversion. On the real side the model predicts that QQE has some stimulative effects on the economy although the real loan rate went up due to QQE intervention discouraging investment.

Table 1: Monetray Pollicy Regimes of BoJ, 1999–2019

Regime	Date	Event
1	1999/02/12	Zero Interest Rate Policy
2	2001/03/19	Traditional QE
3	2006/03/09	Call Rate Target Policy
4-1	2013/04/04	QQE Policy with 2% inflation target
4-2	2016/01/29	QQE with a Negative Interest Rate on Excess Reserve
4-3	2016/09/21	QQE with Yield Curve Control (YCC)

Table 2: Correlations of the seasonally unadjusted monetary base growth with bond yields of varying maturities (1999Q1-2022Q3)

3 year yield -0.30*	4 year yield -0.31*	5 year yield -0.31*
6 year yield -0.30*	7 year yield -0.28*	8 year yield -0.26*
9 year yield -0.24*	10 year yield -0.23*	20 year yield -0.17

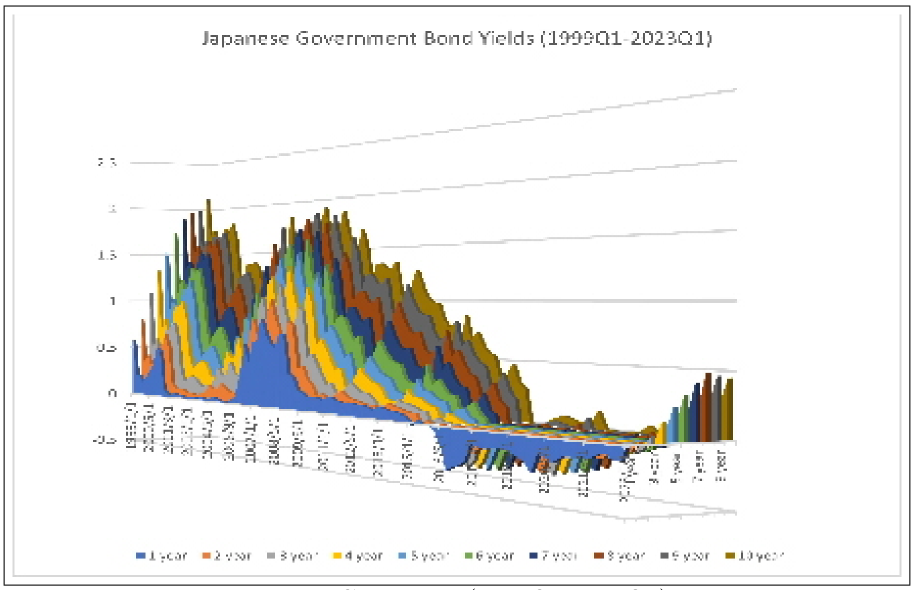


Fig 1: JGB Yields (1999Q1-2023Q1)

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