How Many will Get a Jab, With and Without Herd Immunity?[1]

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Abstract

This paper analyses the reluctance of a significant part of the population to get vaccinated against SARS-CoV-2. We construct a two-period model wherein ex ante identical individuals make voluntary, uncoordinated decisions about social distancing in both periods and vaccination in the second. These decisions affect the prevalence of infection in both periods. The vaccination decision is determined by the individual's immune status from the first period and the interplay between the prevalence of infection and individuals' conjectures about the number choosing vaccination in the second. The analysis covers various scenarios: (i) the existence or otherwise of a threshold at which transmission of infection ceases, (ii) imperfect immunity, and (iii) a sub-population of Covid non-believers. There are diverse subgame perfect Nash equilibria. If zero transmission is not attained in the first period, it will not ensue through vaccination in the second. There can be a mixed strategy configuration wherein some of those who escaped infection in the first period choose vaccination in the second, or even a wholesale rejection of vaccination.

1 Introduction

There have been great efforts worldwide to roll out SARS-CoV-2 vaccination programmes. The vaccines' efficacies vary quite substantially, and their levels seem to be somewhat lower against the delta and, most recently, omicron strains of the virus; but all approved vaccines greatly reduce the chances of hospitalization and death. On the scientific evidence, the probability of adverse side effects is low, albeit these are variable.

What is perhaps surprising is the declared reluctance of a significant part of the population to get vaccinated. Figure 1 depicts the disparate attitudes to vaccination across 15 OECD countries, based on survey data.[1] On December 15, 2021, the declared refusal rate varied from 7% to 25%. According to the UK's Office of National Statistics (2021), the most prominent reason given for refusing vaccination is the anticipated side effects. Yet Graeber et al. (2021) compile evidence from Germany and conclude that only about 70% of the population would opt for a jab, even if there were no such adverse effects. However the latter are assessed, an individual's decision is arguably driven by his or her perception of the likely costs and benefits. The former comprise the certain cost of the treatment itself and the expected costs of its side effects; the benefit stems from the prospective degree of conferred immunity.

[1] The data were collected by YouGov in partnership with the Institute of Global Health Innovation (IGHI) at Imperial College, London. The research covered about 29 countries, interviewing about 21000 people each week. These data are compiled by Our World in Data, available in https://ourworldindata.org/grapher/covid-vaccine-willingness-and-people-vaccinated-by-month&tab=table&country=~FRA

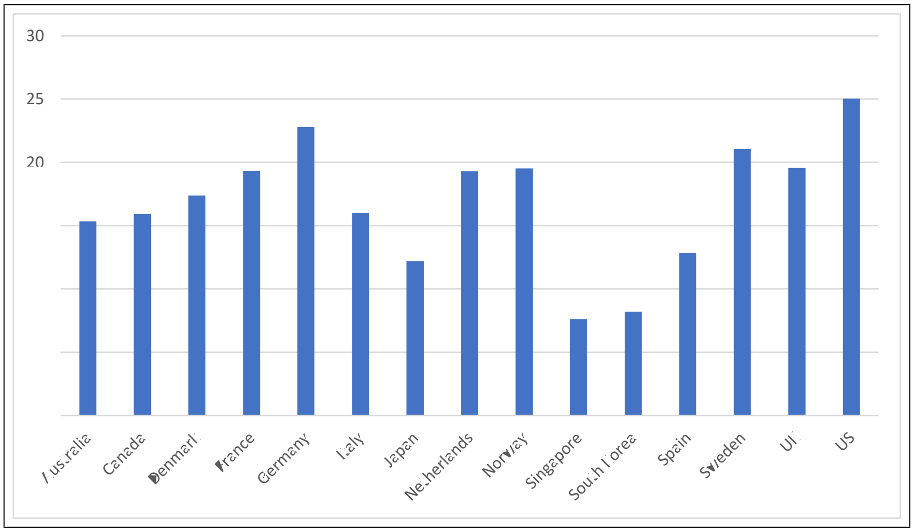

Figure 1: Percent of people unvaccinated and unwilling to get a jab

Declaring an intention to act in a survey is not necessarily the same thing as taking or declining the action itself when the opportunity arises. The said official survey in the UK was conducted in February and March, 2021, just two to three months after the roll out had started. Early in March, 2021, about one third of the population had received at least one jab (Our World in Data, 2022). At this time of writing, 74.2% are fully vaccinated and 58.7% have had at least one booster (*ibid.*). The survey evidence for Germany in Graeber et al. (2021) was published on May 10, 2021. At that time, about one third of the population had received at least one jab. At this time of writing, 77.5% are fully vaccinated – clearly exceeding the declared 70% – and 65.3% have had at least one booster (*ibid.*).

The emphasis on attitudes and intentions rather than actual uptake is indeed a rather striking feature of the literature on 'vaccine hesitancy' related to SARS-CoV-2. This is partly attributable to the lag between roll out and attaining programme maturity in developed countries, whereas roll out itself has been tardy and stuttering in many low- and middle-income countries. Solis Arce et al. (2021), for example, report findings on such hesitancy from studies of 15 countries, the Russian Federation and the U.S. being the sole developed ones. The authors contrast the 'acceptance rates' of 30.4% and 64.6%, respectively, in the latter with an average of 80% among the other 13 countries. Yet according to Our World in Data (2022), the current full vaccination rates for Russia and the U.S. are 51.1% and 67.0%, respectively.[2] That for Pakistan is 55.7%, whose 'acceptance rate' of 66.5% was the lowest among the other 13 countries. Whether the latter shortfall is attributable to the failings of Pakistan's health system rather than a failure of respondents' declared intentions to match their actions is an open question. Other contributions that deal with reported hesitancy include Tran (2021) and Vu (2021) for the U.S. and Unterschultz et al. (2021) for Canada. Vu's study of county-level data is particularly interesting; for he estimates a variety of specifications with both measures of hesitancy and actual vaccination rates (as of September 21, 2021, when the national rate was 57%) as the regressand.[3] What is termed 'rugged individualism' turns out to be a statistically significant obstacle to vaccination in all specifications.

With this general caveat about declared intentions and actual choices, Table 1 reports the correlation between the number of confirmed cases per 100,000 population and the unwillingness to vaccinate for the 15 OECD

[2]The rates for other, selected O.E.C.D. countries are: France, 78.3%; Italy, 80.5%; Japan, 81.0%; South Korea, 86.1%; Spain, 85.5%.

[3]In a recent paper, Daral and Shashidhara (2022) use data from a facebook survey to probe into the reasons for vaccine hesitancy in India. They identify a range of factors, which include insufficient knowledge, doubts about the efficacy of vaccines, concerns about side effects, the desire to wait and watch, an anti-vaccination attitude based on misinformation, and scientifically unfounded beliefs.

countries. What is surprising is a significant positive correlation: in countries where there are proportionally more cases, people are less willing to get vaccinated. This positive correlation holds for various measures of Covid cases, including fatalities. On the other hand, there is also evidence that, well before vaccines became available, people in Western Europe and the US practised social distancing more stringently in order to ward off infection (Maloney and Taskin, 2020). This suggests that the vaccination decision is complicated by the existence of an alternative, independent course of action: agents can use social distancing to lower the chances of infection, an action that also involves externalities. Individuals may form expectations about herd immunity arising in the future, and so choose current social distancing in the light of the contingent distancing and vaccination decisions in the next period. Distancing and vaccination could, therefore, be inter-temporal and strategic in nature.

Table 1: Correlations of vaccine resistance with cases and potential cost of infection

Cumulative cases	Deaths/million	Excess deaths	Serious critical cases
0.54	0.56	0.52	0.55
(0.03)	(0.02)	(0.04)	(0.03)
Source: Our World in Data, Economist (December, 2021), Worldometer			

Certain strategic elements in Covid protection are studied in the literature. Ng (2021) develops a model of mask-wearing in an environment where non-cooperative agents free ride, taking into consideration that a mask benefits others more than it protects the wearer. A similar free-riding issue arises in Basu, Bell and Edwards (2020) in connection with social distancing. Talamas and Vohra (2020) build a network model where agents choose their partners strategically. They conclude that a partially effective vaccine can harm everyone because it increases the chance of risky interactions. We do not do analyse such personal networking. Our focus is primarily on individual choice of vaccination in the presence of strategic complementarity of anonymous social distancing and jab decisions. However, we examine the case wherein vaccination may be partially effective, and in such a scenario of imperfect immunity, we analyze whether there could be a Nash equilibrium wherein some agents choose vaccination.

While there are various contributions on endogenous social distancing,[4] to the best of our knowledge, the strategic elements in vaccination are relatively unexplored in the Covid literature. We develop a simple two-period model to explore such strategic interaction among identical, risk-neutral agents.

There are various subgame perfect Nash equilibria, with symmetric behaviour, fully or among groups, depending on the costs of distancing, the expected costs of infection and the expected costs of vaccination. The latter can be so high as to induce a wholesale rejection of vaccination, with correspondingly extensive social distancing, in the second period, though this outcome seems unlikely in practice.

Much more likely is the possibility that the level of natural immunity yielded by infections in the first period is such that, when combined with conjectures about the number of susceptible individuals who elect to get a

[4] Theoretical studies include Toxvaerd (2020), Eichenbaum et al. (2020), Farboodi et al. (2021), and Getachew (2020); the latter uses a SIR model in a DSGE framework. There is empirical evidence that social distancing is endogenous: see, for example, Chudik et al. (2020) on China.

jab in the second period, all susceptible individuals are indifferent between a jab and taking their chances with optimal social distancing, conditional on the resulting level of immunity in the whole population. A mixed strategy equilibrium can therefore result when a particular fraction of the susceptible population chooses vaccination. That fraction, which is a self-fulfilling expectation in equilibrium, depends crucially on the fraction of the population infected in period 1, which arises from social distancing decisions in period 1. The latter depend, in turn, on the expected cost of infection and that of vaccination; for by conferring full or partial immunity, infection in period 1 will affect the attractiveness of vaccination in period 2. Expectations about the extent of vaccination also depend on the social distancing decisions in period 2 of those infected in period 1, and of those lacking any degree of acquired immunity, as well as the expected costs of infection in period 2.

We analyse two scenarios. The first, which serves as a benchmark, has two salient features. First, both previously infected and vaccinated people are assumed to be immune to symptomatic infection in period 2, and thus do not practise social distancing in that period, although they could still infect others. Second, if the number of such individuals reaches a certain threshold, transmission ceases and the rest of the population are no longer in danger of infection in period 2. In what follows, we call this the zero transmission threshold (or ZTT). It is important to note that this threshold is not the so-called herd immunity threshold (or HIT), where the widely used term 'herd immunity' has a variety of meanings (Fine et al., 2011). In their study of Spain, Garcia-Garcia et al. (2022) define the HIT 'as the minimum proportion of the immune population that will produce a monotonic decrease of new infections.' In the simplest model, the HIT equals $1 - 1/R_0$, where the basic reproduction number R_0 is the number of other individuals each infected person goes on to infect in a population wholly comprised of susceptibles. If the effective reproduction rate, R_e, falls below 1 and remains so, transmission will eventually peter out. Yet there may be numerous new infections in this phase: when HIT = 0.9, 99.6% of the population are ultimately infected (*ibid.*). For the ancestral variant of the virus, the authors' estimates of HIT range from 28.1 to 67.1%; those for the emerging delta variant in 2021 range from 75.1 to 88.8%. With the omicron variant to follow, the corresponding ZTT may well exceed 1.[5]

In the first scenario, the value of the ZTT is assumed, perhaps optimistically, to be less than 1. Wholesale rejection of vaccination can be an equilibrium, but the ZTT is never reached in equilibrium through voluntary vaccination. This finding leaves the possibility of a mixed strategy equilibrium wherein ZTT is not reached. The key results are in keeping with intuition. Higher expected costs of infection reduce the pool of infected people via stronger precautionary social distancing in period 1[6] and so induce a larger proportion of the population to get vaccinated. On the other hand, higher expected costs of vaccination depress social distancing in period 1; for by increasing their chances of infection, individuals improve their chances of avoiding any need to distance or get vaccinated in period 2. There is a data-congruent positive relationship between prevalence and the refusal to get vaccinated.[7]

[5] This is to be interpreted as implying that newly arrived, susceptible individuals would have a strictly positive probability of getting infected.

[6] This is consistent with the empirical finding of Maloney and Taskin (2020) that in the absence of an efficacious vaccine, people distance more when the number of cases is higher.

[7] The case of no herd immunity is a special case, where this threshold level is set at 100% of the population.

In the second, more realistic setting, immunity to symptomatic infection in period 2, whether acquired naturally or through vaccination, is imperfect. All individuals have a strictly positive probability of suffering such an infection in period 2, so that the ZTT exceeds 1. This case is particularly important given the emergence of new variants of the virus, which points to SARS-CoV-2 becoming endemic, much like influenza. Social distancing behaviour in this setting is similar to that in the first, except that all individuals, regardless of their immune status, respond to higher expected costs of infection and vaccination. Quite counter-intuitively, wholly susceptible people may find vaccination more attractive as the associated expected costs rise, because the alternative strategy of simply practising greater social distancing runs into more steeply rising costs at the margin.

Finally, we analyse the effect of the existence of Covid non-believers on the social distancing and vaccination decisions of rational agents. The presence of a group of such non-believers, who practise no social distancing, induces rational agents to distance more and thus lowers infections in that group. Yet this effect is outweighed by the high rate of infection among the non-believers; so that the size of the whole pool of those infected increases with the number of non-believers.

The paper is organized as follows. Section 2 begins by describing the basic setting and the model's structure. The argument then proceeds by backward induction: period 2 in Section 2.1 and period 1 in Section 2.2. The approach to the limit ZTT = 1 is analysed in Section 2.3. An illustrative numerical analysis follows in Section 2.4. Imperfect immunity is treated in Section 3. Non-believers are introduced into the basic setting in Section 4. The main findings are drawn together in Section 5.

2 The Model

At the start of period 1, a population lacking any immunity consists of a continuum of identical, risk-neutral individuals represented by $[0, 1]$. Infection is never fatal.[8] In each period, individuals choose safety measures, which include social distancing and the wearing of masks, taking the choices of other individuals as given. Hereinafter, we call these various measures 'social distancing'. A vaccine will become available at the start of period 2, but its side effects remain uncertain. Those who are susceptible to infection decide whether to get vaccinated.

Individuals are endowed with ω units of a composite good in both periods. Individual i's private cost of social distancing in the measure x_{it} is given by the increasing, strictly convex, twice-differentiable function $c(x_{it})$, where $c(0) = c'(0) = 0$. Infection, if it occurs, inflicts an expected cost of ξ_1, with outcomes that range from few or no symptoms to a stay in an ICU. Those who get infected in period 1 are fully immune in period 2. Denote the set of all such seropositive individuals by S_1, and their share of the population by n_1, which the authorities report at the end of period 1. All individuals have access to free testing at the end of period 1, which offers them a decisive incentive to establish their immune status at the point of deciding about vaccination. Being immune, all individuals in S_1 decline vaccination. Susceptible individuals who choose vaccination also acquire full immunity.[9] All susceptible individuals at the start of period 2 have sharp priors concerning

[8] Deaths in the first period necessarily result in fewer individuals in the second. In fact, the case mortality rate is low, which is not to diminish the tragedies of individual deaths.

[9] Block's assessment (2021) of the evidence is that the immunity conferred by infection is not inferior to vaccination.

the fraction $n_2 \in [0, 1 - n_1]$ who decide to get a jab.[10] Since they are identical *ex ante*, their priors are identical. Let all of them believe that $n_2 = n_2^e$ with probability 1; these vaccinated individuals comprise the set S_2. Although they would suffer no symptoms, all immune individuals can still get infected in period 2 and are then fully infectious.[11] Their social distancing behaviour in period 2, therefore, affects the probability that members of the susceptible, but unvaccinated group, S_3, will get infected.

Index all individuals at the start of period 2 as follows: if $j \in S_1$, then $j \in (1 - n_1, 1]$; if $j \in S_2$, then $j \in (1 - n_1 - n_2, 1 - n_1]$; otherwise, j is susceptible throughout period 2, with $j \in [0, 1 - n_1 - n_2]$. The distancing profile is $\{x_{j2}\}_{j=0}^{j=1}$, the aggregate social distance is $X_2 = \int_0^1 x_{j2} dj$.

The argument proceeds by backward induction. The timing of decisions is as follows. At the start of period 1, each agent forms beliefs about others' distancing decisions in period 1 and the number who will get a jab in period 2. Each individual then chooses some measure of distancing. At the end of period 1, all agents learn their immune status. In period 2, those infected in period 1 (confirmed by free testing) reject both distancing and a jab. In the light of n_1, those who were not infected choose between (i) no jab and taking a chance by distancing and (ii) getting a jab and not distancing.

2.1 Period 2

If $n_1 \geq \underline{n}$ (< 1), then distancing decisions in period alone 1 yield zero transmission in period 2, where $\underline{n}$ denotes the ZTT value. Those who escaped infection in period 1 need have no fears in period 2 and so decline to get vaccinated. With no risk of infection, no one distances in period 2. It remains

[10] For a vigorous argument that rational actors must have sharp priors, see Elga (2010).

[11] The delta and omicrons variant may well have this property.

only to establish under what conditions $\underline{n}$ arises from distancing decisions in period 1, a task that is taken up in Section 2.2.

Suppose, on the contrary, that $\underline{n}$ is not attained in period 1. All individuals who escaped infection in period 1 must decide whether to get a jab. If i in this group decides in favour, she obtains the payoff $\omega - \xi_2$, where ξ_2 is the expected cost of vaccination and assumed to be positive.

If i decides against vaccination, she must choose how much to distance. Let the probability of infection be

$$p_{i2} = f(x_{i2}) \cdot g(X_2, n_1 + n_2, \underline{n}) \text{ if } n_1 + n_2 < \underline{n}; \text{ otherwise, } p_{i2} = 0, \qquad (1)$$

where f is a decreasing, strictly convex and twice-differentiable function, with $f(0) = 1$ and $\lim_{x_{it}\to\infty} f = 0$. Let g be decreasing and continuously differentiable in X_2 and $(n_1 + n_2)$, with $g(0, 0, \underline{n}) = 1$ and $g(0, \underline{n}, \underline{n}) = 0$. Thus, if no one is immune and no one distances, then all get infected. If $n_1+n_2 < \underline{n}$, then given X_2, i can lower her chances of infection by distancing more; and given any $x_{i2} > 0$, p_{i2} goes to zero as $n_1 + n_2$ approaches $\underline{n}$.

The arguments of the function g are features of the environment that influence the effectiveness of i's distancing decision. First, as the aggregate X_2 falls, the environment becomes more hazardous for those lacking immunity, and so increases the probability that they will get infected. Second, and working in the opposite direction, is the fact that, given X_2, as the number of immune individuals (n_1+n_2) rises, the state of zero transmission is approached more closely, thus lowering that probability.[12] Once $n_1 + n_2$ hits the threshold $\underline{n}$, the probability of infection becomes zero.

[12] For given any such pool size, how much they decide, in aggregate, to distance certainly weighs in i's chances of infection. Yet that aggregate depends on both the numbers who can infect i and what they are doing; and these two factors are not perfect substitutes. Thus, the function g also has the argument $n_1 + n_2$.

The first question is, if $\underline{n}$ is not attained in period 1, can it ensue in period 2? We use backward induction to answer this question. Conjectured zero transmission at date 2 means $n_1 + n_2^e \geq \underline{n}$. If, at the start of period 2, i conjectures that at least $\underline{n}$ will be attained, she will decline both vaccination and any distancing ($x_{i2} = 0$), and so obtain ω by free riding. Since agents are identical, everyone not infected in period 1 thinks this way and declines a jab. This wholesale rejection of vaccination implies $n_2 = 0$, which contradicts $n_1 + n_2^e \geq \underline{n}$. We thus have the following proposition.

Proposition 1 *If, under the above assumptions, distancing decisions in period 1 do not yield zero transmission at date 2, then that state cannot arise as a Nash equilibrium from voluntary vaccination in period 2.*

When $\underline{n}$ is not thus attained, individuals know that $n_2 \in [0, \underline{n} - n_1)$. Suppose individual i declines vaccination. She minimises her expected costs by choice of x_{i2}. The associated first-order condition (hereinafter f.o.c.) is

$$f'(x_{i2}) \cdot g \cdot \xi_1 + c'(x_{i2}) = 0. \tag{2}$$

Since both f and c are strictly convex, her optimal choice, x_{i2}^0, is positive and unique for any given X_2 and $n_1 + n_2^e$. In a symmetric configuration, $x_{j2} = x_{i2}^0 \ \forall j \in S_3$ and $X_2 = (1 - n_1 - n_2^e)x_{j2}$, so that x_{j2} satisfies

$$f'(x_{j2}) \cdot g[(1 - n_1 - n_2^e)x_{j2}, n_1 + n_2^e, \underline{n}] \cdot \xi_1 + c'(x_{j2}) = 0 \ \forall j \in S_3. \tag{3}$$

Denote its value by $x_2^0(n_1, n_2^e, \underline{n}, \xi_1)$, which is positive and unique (see Appendix 1, proposition 4). It follows from (1), (2) and (3) that the expected payoff for all $j \in S_3$ is

$$\omega - p_2(x_2^0)\xi_1 - c(x_2^0) = \omega - c(x_2^0) - f(x_2^0)c'(x_2^0)/f'(x_2^0),\text{[13]}$$

[13] The r.h.s. can be written in a form with elasticities: $\omega - c(x_2^0)\left(1 + \frac{\eta_x^c(x_2^0)}{\eta_x^f(x_2^0)}\right)$, where

which they compare with the alternative of vaccination, which yields $\omega - \xi_2$.

The first possibility is that individuals who escaped infection in period 1 reject vaccination in period 2, thus avoiding the cost ξ_2, and expect all others to do likewise ($n_2^e = 0$). If

$$\left\{c(x_2^0) - [f(x_2^0)c'(x_2^0)]/f'(x_2^0)\right\}_{x_2^0 = x_2^0(n_1, n_2^e = 0, \xi_1)} < \xi_2, \tag{4}$$

that expectation is indeed fulfilled under rational choices of distancing, and there is a wholesale rejection of vaccination.

An alternative possibility is that ξ_2 and $n_2^e \in (0, \underline{n} - n_1)$ are such that agents are indifferent between vaccination and distancing, so that (4) holds, instead, as an equality, with $x_2^0(n_1, n_2^e; \underline{n}, \xi_1) > 0$. Then, given ξ_2, $n_2^e \in (0, \underline{n} - n_1)$ must satisfy

$$c(x_2^0) - [f(x_2^0)c'(x_2^0)]/f'(x_2^0) = \xi_2, \tag{5}$$

which involves the interplay of vaccination and distancing decisions.[14] That is to say, given the conjecture in question, all individuals who escaped infection in period 1 will be indifferent between vaccination, which yields the fixed expected pay off $\omega - \xi_2$, and taking their chances with distancing in period 2. For the outcome to be a mixed strategy equilibrium, all individuals who escaped in period 1 must not only conjecture $n_2^e \in (0, \underline{n} - n_1)$, but also choose to get vaccinated with probability $n_2^e/(1 - n_1)$ given that conjecture.

Let $x_2^*(\xi_2)$ satisfy (5) without reference to condition (3). In equilibrium, $x_2^0(n_1, n_2^e, \xi_1) = x_2^*(\xi_2)$ (see Appendix 1 for the details). As ξ_2 varies, then

$\eta_x^c(x_2^0) = c'(x_2^0)x_2^0/c(x_2^0)$ and $\eta_x^f(x_2^0) = f'(x_0^2)x_2^0/f(x_2^0)$ are the respective elasticities of private cost and private benefit, in the form of a reduction in the probability of suffering a symptomatic infection, with respect to social distancing.

[14] As n_2^e increases, $x_2^0(n_1, n_2^e, \xi_1)$ will decrease steadily (see Appendix 1), so that if ξ_2 is not too large, there is an $n_2^e \in (0, \underline{n} - n_1)$ such that $x_2^0(n_1, n_2^e, \xi_1)$ also satisfies (5).

so, too, does the equilibrium pair $(n_2^e, x_2^*(\xi_2))$, with $n_2^e > 0$ so long as ξ_2 is not so large that condition (4) holds.

The foregoing results are summarised as

Proposition 2 *Suppose zero transmission does not arise from infections in period 1.*
(i) If condition (4) holds, universal rejection of vaccination is a Nash equilibrium. If g satisfies condition (22) in Appendix 1, then it is the only Nash equilibrium.
(ii) If the converse of condition (4) holds, ξ_2 is sufficiently small and (22) holds, then there is a mixed strategy Nash equilibrium wherein the share of those susceptible at the start of period 2 who choose vaccination is $n_2^e/(1 - n_1)$, where $n_2^e \in (0, \underline{n} - n_1)$ is such that x_2^0 satisfies (5).

The burden of scientific evidence is that the expected cost of vaccination is much less than that of relying on privately optimal distancing, so that condition (4) is violated. The indifference condition (5) is quite realistic, for it is common to observe unvaccinated sub-populations when vaccination is voluntary. Where SARS-CoV-2 is concerned, this may not be due wholly to people's ignorance, or dismissal, of scientific knowledge. It could be based on their social distancing behaviour in period 1, the resulting number of infected people, n_1, and conjectures about the vaccination share n_2^e.

2.2 Period 1

In view of proposition 1, the first question to be addressed is whether distancing decisions in period 1 yield zero transmission in period 2, and thus make vaccination and distancing in that period superfluous. If X_1 is such that this would be the outcome, all individuals know that they will obtain

ω for sure in period 2, so that the structure reduces to a one-shot game. Individual i's f.o.c. is then $f'(x_{i1})g(X_1, 0, 1)\xi_1 + c'(x_{i1}) = 0$. Assuming a symmetric equilibrium, let $x_{j1} = x_1^*(\underline{n}, \xi_1)\ \forall j$, where $x_1^*(\underline{n}, \xi_1)$ satisfies $f'(x_1^*(\underline{n}, \xi_1))g(x_1^*(\underline{n}, \xi_1), 0, 1)\xi_1 + c'(x_1^*(\underline{n}, \xi_1)) = 0$. Now, in any symmetric allocation $x_{j1} = x_1^*\ \forall j$, the law of large numbers yields

$$n_1(x_1^*) = p_{j1} = f(x_1^*) \cdot g(x_1^*, 0, 1)\ \forall j. \tag{6}$$

If $n_1(x_1^*(\underline{n}, \xi_1)) \geq \underline{n}$, then there is a subgame perfect Nash equilibrium wherein $x_{j1} = x_1^*(\underline{n}, \xi_1)\ \forall j$ and there is neither distancing nor vaccination in period 2. If, on the contrary, zero transmission does not thus ensue, then, in virtue of proposition 1, it will not do so from voluntary vaccination in period 2, unless the expected costs of vaccination are zero, in which event all individuals can obtain ω in that period by choosing vaccination, thus yielding $n_1(x_1^*) + n_2 = 1 > \underline{n}$. This argument establishes

Proposition 3 *If $n_1(x_1^*(\underline{n}, \xi_1)) \geq \underline{n}$, then there is a subgame perfect Nash equilibrium wherein zero transmission is attained as a result of distancing decisions in period 1. If $n_1(x_1^*(\underline{n}, \xi_1)) < \underline{n}$, then there is a subgame perfect Nash equilibrium wherein zero transmission is attained through voluntary vaccination in period 2 if and only if $\xi_2 = 0$.*

Remark. If the threshold is attained, $n_1(x_1^*(\underline{n}, \xi_1))$ is independent of ξ_2. If the threshold is not attained, recall that $\xi_2 > 0$ is a necessary condition for proposition 1 to hold.

If the threshold is not attained and $n_2^e > 0$, all those who escaped infection in period 1 will obtain the pay off $\omega - \xi_2$ n period 2, being indifferent between vaccination and distancing. Knowing this, all individuals obtain, evaluated *ex ante* in period 1, the expected pay off $\omega - (1 - p_1(x_1^*))\xi_2$ in the symmetric equilibrium in period 2, and the expected pay off

$$\omega - p_1(x_1^*)\xi_1 - c(x_1^*) = \omega - f(x_1^*) \cdot g(x_1^*, 0, 1)\xi_1 - c(x_1^*)$$

in period 1. Note that the expected cost of vaccination *ex ante* is net of the probability of getting infected in period 1, infection making vaccination unnecessary. The resulting value of the individual preference functional V_i is

$$V_i = (1+\beta)\omega - (p_1\xi_1 + c(x_1^*)) - \beta(1 - p_1(x_1^*))\xi_2 \;\forall i,$$

where β (≤ 1) is the discount factor. Suppose, in keeping with the evidence, $\xi_1 > \beta\xi_2$.

Proposition 4 *If there exist*
(i) an x_1^* *such that* $f'(x_1^*)g(x_1^*, 0, 1)(\xi_1 - \beta\xi_2) + c'(x_1^*) = 0$ *and* $f(x_1^*)g(x_1^*, 0, 1) = n_1 < \underline{n}$*, and*
(ii) an $n_2^e \in (0, \underline{n} - n_1)$ *that satisfies condition (3) when* $x_{j2} = x_2^*(\xi_2)$ *and the indifference condition (5),*
then the allocation $x_{j1} = x_1^* \;\forall j$*,* n_2^e*,* $x_{j2} = x_2^*(\xi_2) \;\forall j$ *is a subgame perfect Nash equilibrium, wherein all susceptible individuals at the start of period 2 choose vaccination with probability* $n_2^e/(1 - n_1)$*.*

Proof: see Appendix 1, which also addresses the question of uniqueness.

The condition $f'(x_1^*)g(x_1^*, 0, 1)(\xi_1 - \beta\xi_2) + c'(x_1^*) = 0$ implies that, in keeping with intuition, x_1^* is increasing in ξ_1 and decreasing in ξ_2; for distancing more in period 1 lowers the chances of infection in that period, but increases the chances of entering the next without immunity, and thus having to face the vaccination decision. If ξ_2 increases, then so will n_1, because x_1 decreases (see eq. (6)).

2.3 The probability of transmission is always positive

It is quite possible that $\underline{n} \geq 1$, so that there is only individual immunity. It is straightforward to show how, in a mixed strategy equilibrium, agents respond to changes in that threshold leading up to the value 1. This is accomplished by differentiating (10) totally, while noting that x_1^* and x_2^* are independent of $\underline{n}$ when $n_1 < \underline{n}$. Some manipulation yields

$$\frac{\underline{n}}{n_2^e} \cdot \frac{dn_2^e}{d\underline{n}} = \frac{n_1 + n_2^e}{\underline{n}} \cdot \frac{a(1 - n_1 - n_2^e)x_2^* + 1}{a(1 - \underline{n})x_2^* + 1}.$$

The resulting (left-hand) expression at $\underline{n} = 1$ reduces to:

$$\frac{\underline{n}}{n_2^e} \cdot \frac{dn_2^e}{d\underline{n}} = (n_1 + n_2^e)[a(1 - n_1 - n_2^e)x_2^* + 1].$$

In such configurations, therefore, the receding prospect of attaining zero transmission encourages vaccination all the way to $n_1 = 1$. Given the functional forms in Section 2.4, it is seen from (8) and (9) below that changes in $\underline{n}$ have no effect on the number of infections in period 1.

2.4 Quantitative Analysis

Specific functional forms are needed. Let $c(x) = x^2/2$ and

$$p_{i2} = \left(1 - \frac{n_1 + n_2}{\underline{n}}\right)[(x_{i2} + 1)(aX_2 + 1)]^{-1} \text{ if } n_1 + n_2 < \underline{n}; \text{ otherwise, } p_{i2} = 0, \tag{7}$$

where $a > 0$ is a constant.[15] In a symmetric, mixed strategy equilibrium, x_1^*, n_1, x_2^*, and n_2^e are then related by the following equation system. The conditions of part (i) in proposition 4 specialise to

$$x_1^*(x_1^* + 1)^2(ax_1^* + 1) = \xi_1 - \beta\xi_2 \tag{8}$$

[15]Observe that $\eta_x^c = 2$ and $|\eta_x^f| = x_{it}/(x_{it} + 1)$, so that $\eta_x^c/|\eta_x^f|$ can take any value in $(2, \infty)$ and $c(x)(1 + \eta_x^c/|\eta_x^f|)$ any value in $[0, \infty)$.

and

$$n_1 = [(x_1^* + 1)(ax_1^* + 1)]^{-1}, \tag{9}$$

respectively. Eq. (8) has a unique positive solution. If $n_1 < \underline{n}$, it is admissible. Eq.(3) becomes

$$x_2^*(x_2^* + 1)^2(a(1 - n_1 - n_2^e)x_2^* + 1) = \left(1 - \frac{(n_1 + n_2^e)}{\underline{n}}\right)\xi_1. \tag{10}$$

Hence the indifference condition (5) reduces to

$$x_2^*(1 + 3x_2^*/2) = \xi_2, \tag{11}$$

which has the unique positive solution $x_2^*(\xi_2) = [-1 + (6\xi_2 + 1)^{1/2}]/3$. Eqs. (8) and (10) can then be solved for x_1^* and n_2^e. Since $\partial g/\partial n_2 = -[a(1 - \underline{n})x_2^* + 1]/[\underline{n}(ax_2^* + 1)^2] < 0$, n_2^e is unique. Finally, (9) yields n_1.

Figures 2 and 3 report the sensitivity of the key variables to changes in the expected cost parameters ξ_1 and ξ_2, respectively. We assume that the expected cost of vaccination is at most one quarter, say, of the expected cost of an infection. As baseline, let $\xi_1 = 3$, $\xi_2 = 0.75$, $a = 2$ and $\beta = 1$. Drawing on Garcia-Garcia et al. (2022), choose the rather optimistic threshold value $\underline{n} = 0.8$. The level of distancing at date 1, x_1^*, rises with the expected cost of infection (see eq. (8)); as noted above, the latter has no effect on distancing in period 2. The number of those infected, n_1, falls, as seen from (9), while the conjectured number receiving a jab, n_2^e, rises. The probability that susceptible individuals will get a jab is around 0.35 and rises with ξ_1.

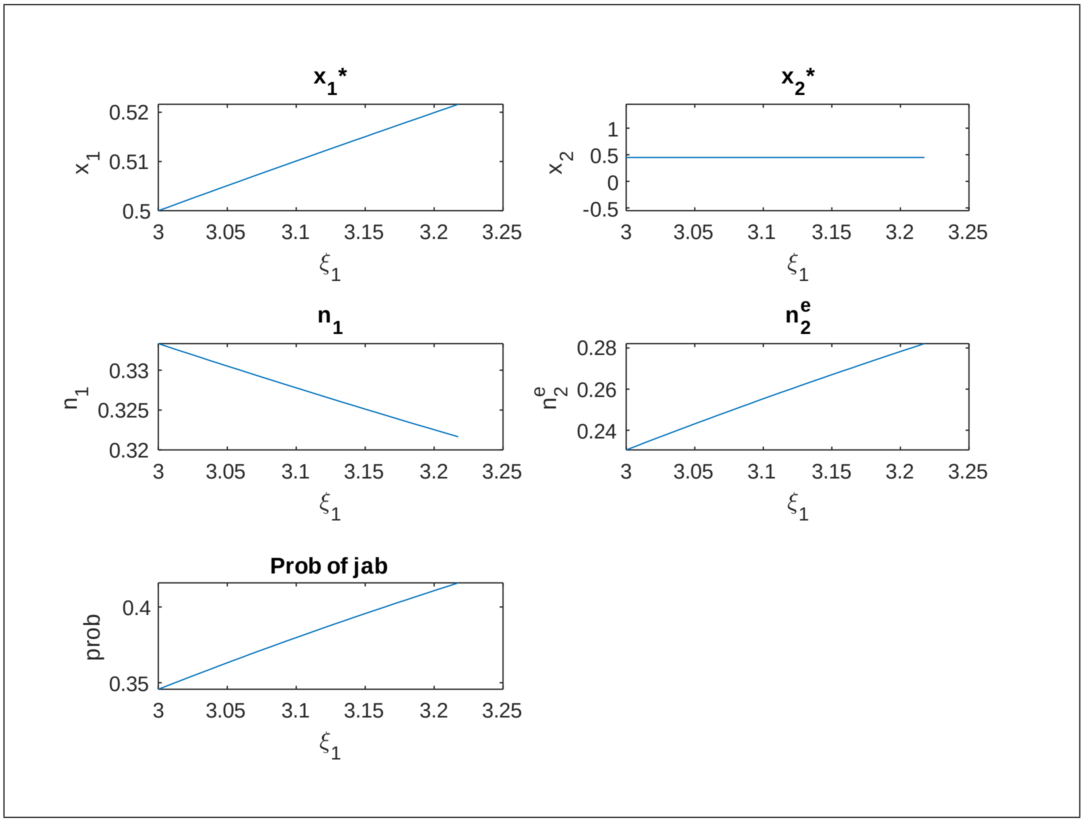

Figure 2: Effect of a rise in expected cost of infection

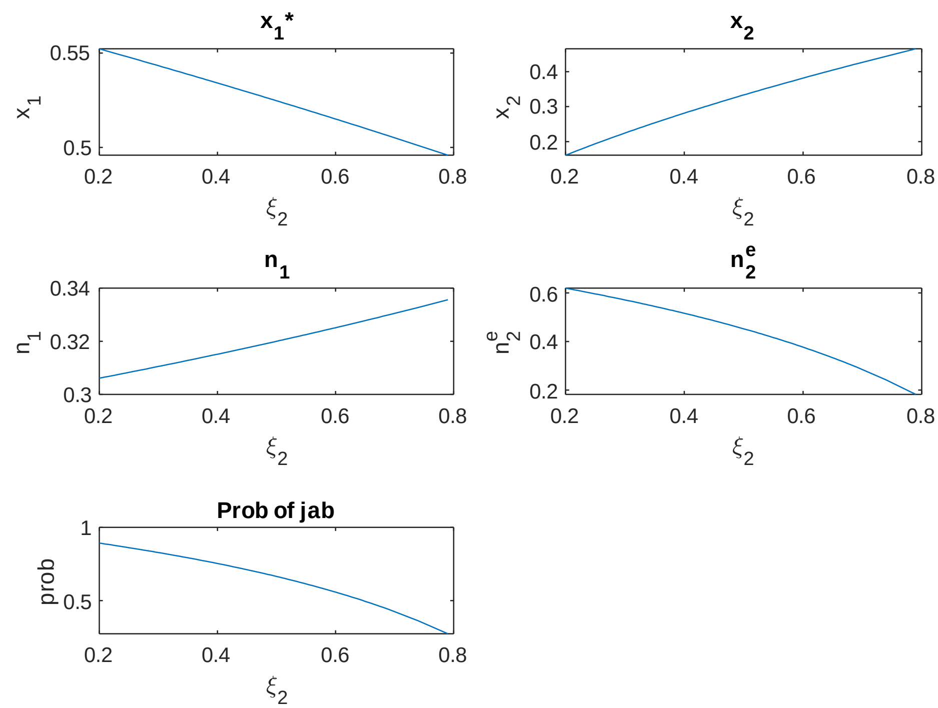

Figure 3: Effect of a rise in expected cost of vaccines.

The responses to a change in the expected cost of vaccination, ξ_2, run in the opposite direction (see Figure 3). As this cost rises, people distance less at date 1 and more at date 2. By increasing their chances of getting infected at date 1, agents lower their chances of incurring the expected cost of vaccination at date 2. Individuals expect fewer people to get vaccinated, and this lowers the probability of vaccination, since the number of those susceptible at date 2 falls only slightly as ξ_2 rises. When $\xi_2 = 0.83$, all those not infected in period 1 reject vaccination.[16]

[16]It is instructive note that the inverse relationship between n_2^e and ξ_2 holds for all ξ_1 such that $\xi_1 > ax_2^{*2}(x_2^* + 1)^2/\underline{n}$. This can be verified by differentiating (10) w.r.t. ξ_2 and

3 Imperfect Immunity

Naturally acquired immunity to certain diseases is perfect, but that rarely holds for vaccination. Neither holds for SARS-CoV-2, even though previous infection or vaccination greatly reduces the chances of subsequent symptomatic infection.

Suppose there is a pool of infectious individuals at the start of period 2. Imperfect immunity carries the implication that even if all individuals either had been infected in period 1 or get vaccinated in period 2, each and every one of them would have a strictly positive probability, quite possibly small, of suffering symptomatic infection in period 2. The law of large numbers implies that the number of infections in that period will be strictly positive and in excess of the number in the said pool.[17]

We postulate that Covid will become endemic, and always actively present, like influenza.[18] Whether an individual gets infected in period 2 then depends on three factors: the environment, as summarized by the aggregate X_2; the individual's choice of distancing, x_{i2}; and the individual's immune status, whether from infection in period 1, vaccination in period 2, or neither.

exploiting (8).

[17] This is certainly false for measles, small outbreaks of which occur in rich countries only when unvaccinated children from poorer ones turn up in some numbers in a particular city, wherein some of the locals had decided not to have their children vaccinated against the disease. That occurred in Berlin in 2015, the children arriving from the Balkans. Those who contract measles enjoy lifetime immunity.

[18] This is particularly true with the emergence of new variants such as delta and omicron, against which vaccines are less efficacious.

3.1 Period 2

No one is completely immune, which means seropositive (S_1), vaccinated (S_2) and susceptible (S_3) individuals can all suffer symptomatic infection, though their respective probabilities of doing so will differ. The probability function in period 1 stays the same, but those for period 2 become

$$p_{i2} = p(k)\left[(x_{i2}+1)(aX_2+1)\right]^{-1}, \forall i \in S_k,\ k = 1,2,3, \qquad (12)$$

where the parameter $p(k)$ reflects the immune status of group k. Let $p(1) = p(2) < p(3)$.

The expected costs of infection also differ. In period 1, when all are susceptible, let $\xi_1 = \xi_1(0)$. In period 2, $\xi_1(1) = \xi_1(2) < \xi_1(3)$, where it is possible that $\xi_1(3)$ differs from $\xi_1(0)$, since treatment may improve through experience.

Proceeding as before, individuals' decisions in period 2 depend on their immune status. Let $c = x_{i2}^2/2$. Then the f.o.c. for distancing is

$$p(k)\xi_1(k) = x_{i2}(x_{i2}+1)^2(aX_2+1),\ i \in S_k,\ k = 1,2,3. \qquad (13)$$

It is seen that the optimum now involves strictly positive distancing for any X_2, and that the optimal level is unique for each group. For those who choose vaccination, there is the fixed cost ξ_2. By assumption, $p(1) = p(2) \equiv \widetilde{p}$ and $\xi_1(1) = \xi_1(2) = \widetilde{\xi}_1$, so that $x_{i2}^0(1) = x_{i2}^0(2)$. For those who decline to get vaccinated, $p(3) > \widetilde{p}$ and $\xi_1(3) > \widetilde{\xi}_1$, but without the burden of the expected cost ξ_2. Thus, for any X_2, members of S_3 will distance more.

We focus on mixed strategy equilibria, though it should be noted that for some constellations of parameters, there will be a wholesale rejection of vaccination. For others, in contrast to perfect immunity, wholesale acceptance can be an equilibrium (see Appendix 2).

In a symmetric equilibrium, $X_2 = (n_1 + n_2)x_2^0(2) + (1 - n_1 - n_2)x_2^0(3)$. Substituting into (13), we obtain two equations in $x_2^0(2)$ and $x_2^0(3)$:

$$p(k)\xi_1(k) = x_2^0(k)(x_2^0(k)+1)^2[a((n_1+n_2)x_2^0(2)+(1-n_1-n_2)x_2^0(3))+1], \; k = 2, 3. \tag{14}$$

Those not infected in period 1 will be indifferent between vaccination and taking their chances in period 2. Proceeding as Section 2.1, and noting (5), we obtain the corresponding indifference condition

$$x_2^0(3)[1 + 3x_2^0(3)/2] - x_2^0(2)[1 + 3x_2^0(2)/2] = \xi_2, \tag{15}$$

which implies that $x_2^0(3) > x_2^0(2)$. From (14),

$$\frac{x_2^0(2)(x_2^0(2) + 1)^2}{x_2^0(3)(x_2^0(3) + 1)^2} = \frac{p(2)\xi_1(2)}{p(3)\xi_1(3)} \equiv \rho, \tag{16}$$

where the constant ρ is surely much smaller than 1, implying that $x_2^0(3)$ greatly exceeds $x_2^0(2)$.[19] It also follows that $x_2^0(2)$ and $x_2^0(3)$ must move together in response to changes in other variables. In particular, they are increasing in ξ_2 (see Appendix 2). Since the function $c(x_{it})$ is strictly convex, the marginal cost of social distancing rises much more steeply for the unvaccinated, thus making vaccination relatively more attractive.

3.2 Period 1

At the start of period 1, i chooses x_{i1} so as to maximise

$$\begin{aligned} V_i &= (1 + \beta)\omega - p_{i1}\xi_1(0) - x_{i1}^2/2 - \beta(x_2^0(2))^2/2 - \beta(1 - p_{i1})\xi_2, \\ &= \{(1 + \beta)\omega - \beta[(x_2^0(2))^2/2 + \xi_2]\} - p_{i1}[\xi_1(0) - \beta\xi_2] - x_{i1}^2/2, \end{aligned} \tag{17}$$

[19] Total differentiation also yields $dx_2^0(3)/dx_2^0(2) > 1$ if and only if $1 + x_2^0(3) + x_2^0(3) > 3x_2^0(2)x_2^0(3)$.

where the expression in braces is parametric for i. The associated f.o.c. is

$$p(0)[\xi_1(0) - \beta\xi_2] = x_{i1}(x_{i1} + 1)^2(aX_1 + 1), \ \forall i, \tag{18}$$

where it is assumed, plausibly, that $\xi_1(0) > \beta\xi_2$. In a symmetric equilibrium, $x_{i1} = x_1^* \, \forall i$, so that

$$p(0)[\xi_1(0) - \beta\xi_2] = x_1^*(x_1^* + 1)^2(ax_1^* + 1), \tag{19}$$

which has a unique positive solution. By the law of large numbers,

$$n_1(x_1^*) = p_1(x_1^*) = p(0)/[(x_1^* + 1)(ax_1^* + 1)]. \tag{20}$$

As in Section 2, $n_1(x_1^*)$ is decreasing in $\xi_1(0)$ and increasing in ξ_2. This argument establishes the counterpart of Proposition 4:

Proposition 5 *Let x_1^* satisfy (19). If there is an $n_2^e = n_2 \in (0, 1 - n_1(x_1^*))$ such that $x_2^0(1) = x_2^0(2)$ and $x_2^0(3)$ satisfy conditions (13) and (15), then the allocation*

$$x_{j1} = x_1^* \, \forall j, \ n_2^e, \ x_{j2} = x_2^0(2) \, \forall j \in S_1 \cup S_2, \ x_{j2} = x_2^0(3) \, \forall j \in S_3$$

is a subgame perfect Nash equilibrium, wherein all individuals lacking any immunity at the start of period 2 elect to get vaccinated with probability $n_2^e/(1 - n_1(x_1^*))$.

In view of the argument at the close of Section 3.1, an increase in ξ_2 may induce an increase in n_2^e, despite always inducing an increase in $n_1(x_1^*)$. The numerical examples in Appendix 2 demonstrate this possibility.

4 Non-Believers

Thus far, all agents are rational, possessing common prior beliefs. Now suppose there is a group of Covid non-believers, S_0, comprising the share λ of the population. Thus persuaded, they neither practise social distancing nor opt for a jab.[20]

To illustrate the effects of this group on the choices of rational agents, we modify the probability functions in Section 2 as follows:

$$p_{i1} = f(x_{i1}) \cdot g[(1-\lambda)X_1, 0, 1]$$

$$p_{i2} = f(x_{i2}) \cdot g[(1-\lambda)X_2, n_1+n_2, \underline{n}] \text{ if } n_1+n_2 < \underline{n}; \text{ otherwise, } p_{i2} = 0.$$

whereby non-believers choose $x_{01} = x_{02} = 0$. We concentrate on mixed strategy equilibria. Consider one such equilibrium well in the interior of the subset of admissible parameter values in Section 2, wherein all agents are rational ($\lambda = 0$). Suppose the group of non-believers is sufficiently small to ensure the existence of a like equilibrium in this changed setting.

If $x_{j1} = x_1^* \; \forall j \neq i$ (i and all j rational), individual i's f.o.c. is

$$f'(x_{i1})g[(1-\lambda)x_1^*, 0, 1](\xi_1 - \beta\xi_2) + c'(x_{i1}) \leq 0, \; x_{i1} \geq 0.$$

It follows at once that, in a symmetric allocation, rational individuals distance more in period 1 as the non-believers become more numerous. Let $n_1(x_1^*, \lambda)$ denote the size of the pool of all naturally immune individuals, n_1^r that of those who are rational.

[20] There are also people who, for religious or other reasons, categorically reject vaccination, but not the array of measures called 'distancing'. Their presence in the population further complicates the analysis; for their choice of distancing will generally differ from that of all rational individuals in period 1 and from that of rational individuals who decide against vaccination in period 2. We do not pursue this extension here.

The argument thus far establishes a modified form of proposition 4.

Proposition 6 *Let non-believers comprise the fraction λ of the whole population. If there exist*
(i) an x_1^* such that $f'(x_1^*)g[(1-\lambda)x_1^*, 0, 1](\xi_1 - \beta\xi_2) + c'(x_1^*) = 0$, an $n_1(x_1^*, \lambda) < \underline{n}$, and
(ii) an $n_2^e \in (0, \underline{n} - n_1)$ that satisfies condition (3) when $x_{j2} = x_2^*(\xi_2)\ \forall j \in S_3$, and the indifference condition (5), then the allocation

$$x_{j1} = x_{j2} = 0\ \forall j \in S_0,\ x_{j1} = x_1^*\ \forall j \notin S_0,\ n_2^e,\ x_{j2} = x_2^*(\xi_2)\ \forall j \in S_3$$

is a subgame perfect Nash equilibrium, wherein all rational susceptible individuals at the start of period 2 elect to get vaccinated with probability $n_2^e/(1 - n_1^r)$.

It is proved in Appendix 1 that, with the specific forms of c and p_{it} in Section 2.4,

$$n_1(x_1^*, \lambda) = \frac{x_1^*(1 + \lambda x_1^*)(x_1^* + 1)}{\xi_1 - \beta\xi_2}, \tag{21}$$

where x_1^* satisfies the f.o.c.

$$x_1^*(x_1^* + 1)^2[(a(1 - \lambda)x_1^* + 1] = \xi_1 - \beta\xi_2$$

and the contribution of rational individuals to that pool is

$$n_1^r = (1 - \lambda)[(x_1^* + 1)(a(1 - \lambda)x_1^* + 1)]^{-1}.$$

It follows from (21) that $n_1(x_1^*, \lambda)$ is increasing in λ; for $x_1^*(1 + \lambda x_1^*)(x_1^* + 1)$ is increasing in x_1^* and x_1^* is increasing in λ. Total differentiation of the f.o.c., with $\xi_1 - \beta\xi_2$ held constant, yields the closed form

$$\frac{dx_1^*}{d\lambda} = \frac{(x_1^*)^2(x_1^* + 1)}{1 + (1 + 3a)x_1^* + a(3 - 2\lambda)(x_1^*)^2} > 0.$$

Yet although rational individuals distance more as the number of non-believers increases, the direct effect of zero-distancing among the latter overwhelms the defensive efforts of the former where $n_1(x_1^*, \lambda)$ is concerned.

As for the behaviour of n_2^e, the findings in Section 2.4 rather point to an ambiguous response to changes in λ (see Appendix 1 for the details).

The comparative statics results are summarised as follows.

Proposition 7 *Under the conditions of proposition 6, an increase in the proportion of non-believers in the population has the following effects.*
(i) Rational individuals distance more in period 1.
(ii) The proportion of the population infected in period 1 increases.
(iii) The uptake of vaccination among susceptible rational individuals may rise or fall.

5 Conclusions

This paper deals with a two-period setting in which individuals make wholly voluntary, uncoordinated decisions about social distancing and vaccination, where the latter carries the risk of side effects. Both actions affect the prevalence of symptomatic infection.

There exist various subgame perfect Nash equilibria. If individual immunity is perfect and the zero transmission threshold is less than 1, then zero transmission in the second period can arise from distancing decisions in the first period. If it does not thus arise, it can never do so as a result of vaccination at the start of the second. There can be a mixed strategy equilibrium, wherein some of those who escaped infection in the first period choose vaccination in the second, or even a wholesale rejection of vaccination, depending on the expected costs of infection and those of vaccination.

These outcomes among rational agents also hold in the presence of a sufficiently small group of Covid deniers. If individual immunity is imperfect, all those not infected in the first period may indeed choose vaccination in the second.

Lower perceived expected costs of vaccination induce steady movements towards the state of zero transmission. If, in the limit, the expected cost of vaccination were zero, then vaccination of all those who escaped infection in period 1 would be a focal conjecture. A lump sum tax to cover both the expected cost and a small net inducement for individuals to turn up for their jab would then yield the state in question.

When rigorous enforcement of distancing and universal, mandatory vaccination are out of the question, there remains persuasion. An information campaign whose tone is reasoned and encouraging, rather than hectoring through a megaphone, may nudge the whole population towards substantial immunity. This outcome is all the more likely to result when naturally acquired immunity is duly recognised.

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Appendix 1

Full immunity: distancing and vaccination decisions in period 2

Claim: x_2^0 is continuously differentiable in n_2^e; it is continuously decreasing in n_2^e if and only if

$$\partial g/\partial n_2^+ - x_2^0 \cdot \partial g/\partial X_2 < 0, \tag{22}$$

where $n_2^+ \equiv n_1 + n_2$. Let g be such that condition (22) holds everywhere. That is to say, a marginal increase in the share of all immune individuals reduces g by an amount that outweighs the increase induced by the associated changes in distancing.[21] This holds when those who reject vaccination choose sufficiently limited distancing or $|\partial g/\partial X_2|$ is sufficiently small.

Denote the partial derivatives of g by g_{X_2} and $g_{n_2^+}$, respectively. Differentiating (3) totally, rearranging and noting that n_1 is fixed in period 2, we obtain

$$\frac{dx_2^0}{dn_2^e} = \frac{-x_2^0 g_{X_2} + g_{n_2^+}}{(f'' g \cdot \xi_1 + c'') + (1 - n_1 - n_2^e) g_{X_2} f' \cdot \xi_1},$$

where the assumptions on f, c and g include $f' < 0, f'' > 0, c'' > 0, g_{X_2} < 0$ and $g_{n_2^+} < 0$. Thus, the denominator is positive, so that x_2^0 decreases or increases as n_2^e increases according as $-x_2^0 g_{X_2} + g_{n_2^+}$ is negative or positive. The assumptions on c, f and g are such that both the numerator and the denominator are continuous, and since the denominator nowhere vanishes, x_2^0 decreases or increases continuously as n_2^e increases. ■

Claim: given c and f, distancing in period 2 depends only on ξ_2.

By assumption, $c(0) = c'(0) = 0$, $c(x_2)$ increases without bound as x_2 increases, and $-f/f' > 0$. Hence, there exists an $x_2 > 0$ such that, for any

[21] In elasticity form, this condition can be expressed as $\eta(g)_{n_2^+} - [n_2^+/(1-n_2^+)]\eta(g)_{X_2} < 0$.

$\xi_2 > 0$,

$$c(x_2) - [f(x_2)c'(x_2)]/f'(x_2) = \xi_2.$$

Denote that value by $x_2^*(\xi_2)$, which is increasing in ξ_2 and independent of ξ_1, as well as of $\underline{n}, n_1$ and n_2^e for all $n_2^e \in (0, \underline{n} - n_1)$, the latter being arguments only of g. Since (5) holds, $x_2^*(\xi_2) = x_2^0(n_1, n_2^e, \xi_1)$ and $c(x_2^*(\xi_2)) + p_2(x_2^*(\xi_2))\xi_1 = \xi_2$. ■

Vaccination: expected costs and acceptance

Suppose the threshold is not attained and that ξ_2 is close to zero, so that condition (4) does not hold. Then considerations of continuity suggest that there exists a subgame perfect Nash equilibrium wherein $n_1 < \underline{n}$ and $n_1 + n_2$ is smaller than, but close to, $\underline{n}$. In such an equilibrium, all individuals know that if they get infected in period 1, they will obtain ω for sure in period 2, whereas if they escape, they will obtain the expected pay off $\omega - \xi_2$. If ξ_2 is sufficiently large, however, condition (4) will hold. This indicates that as the expected cost ξ_2 increases steadily from zero, it will induce the level of voluntary vaccinations to fall steadily, eventually to zero, with accompanying adjustments to the level of distancing in period 2.

Proof of proposition 4

In view of

$$V_i = (1+\beta)\omega - (p_{i1}\xi_1 + c(x_{i1})) - \beta(1 - p_{i1}(x_{i1}))\xi_2,$$

let $x_{i1}^0(x_1^*)$ minimize the (convex) function $f(x_{i1})\cdot g(x_1^*, 0, 1)(\xi_1 - \beta\xi_2) + c(x_{i1})$. Then $x_{i1}^0(x_1^*)$ satisfies

$$f'(x_{i1})g(x_1^*, 0, 1)(\xi_1 - \beta\xi_2) + c'(x_{i1}) \leq 0, \; x_{i1} \geq 0. \tag{23}$$

It is seen that $x^0_{i1}(x^*_1)$ is unique and positive, and that it depends only on the difference $\xi_1 - \beta\xi_2$. Individual i will deviate from the hypothesized symmetric equilibrium in period 1 if and only if

$$p^0_{i1}(x^*_1)(\xi_1 - \beta\xi_2) + c(x^0_{i1}(x^*_1)) < p^*_1(\xi_1 - \beta\xi_2) + c(x^*_1). \tag{24}$$

Observing that, in the limit, condition (24) is violated when the strict equality

$$p^0_{i1}(x^*_1)(\xi_1 - \beta\xi_2) + c(x^0_{i1}(x^*_1)) = p^*_1(\xi_1 - \beta\xi_2) + c(x^*_1),$$

holds, i.e., no deviation, we have established the conditions in part (i). ■

If $\xi_1 \leq \beta\xi_2$, i will choose $x_{i1} = 0$, as will everyone else, so that $n_1 = 1$, contradicting the conjecture $n^e_2 \in (0, \underline{n})$. This obvious result is an instance of the first part of proposition 3.

On the uniqueness of (symmetric) equilibrium, condition (23) becomes

$$f'(x^*_1)g(x^*_1, 0, 1)(\xi_1 - \beta\xi_2) + c'(x^*_1) = 0.$$

This equation has a unique positive solution, and hence the pair $(x^*_1, n_1(x^*_1))$ is unique. If $n_1(x^*_1) < \underline{n}$, the solution is admissible. Turning to $x^*_2(\xi_2)$, this is always unique. Given that value, n_2 is unique if the function g is monotonically decreasing in n_2, whereby g is positive when $n_2 = 0$ and g vanishes when $n_1 + n_2 = \underline{n}$.

Imperfect immunity: vaccination decisions

Suppose $n_2 = 0$. Then $X_2 = n_1 x^0_2(2; n_2 = 0) + (1 - n_1)x^0_2(3; n_2 = 0)$, where the levels of distancing will increase with $1 - n_1$, the size of the pool of those lacking any immunity. If

$$x^0_2(2; n_1, n^e_2 = 0)[1 + 3x^0_2(2; n_1, n^e_2 = 0)/2] < \xi_2,$$

all will reject vaccination, now with members of S_1 distancing somewhat in view of their imperfect immunity.

At the other extreme, suppose all individuals but i have some immunity, i.e., $n_1 + n_2 = 1$, individuals having no measure. Then, in a symmetric equilibrium, $X_2 = x_2^0(2; n_1 + n_2 = 1)$. Substituting into (13), we obtain

$$p(2)\xi_1(2) = x_{j2}^0(x_{j2}^0 + 1)^2(ax_2^0(2) + 1), \ \forall j \neq i.$$

Imposing symmetry yields a quartic, which possesses a positive solution $x_2^0(2; n_1 + n_2 = 1)$. Now suppose i decides against vaccination. From (13), we obtain the cubic

$$x_{i2}(x_{i2} + 1)^2 - p(3)\xi_1(3)/[ax_2^0(2; n_1 + n_2 = 1) + 1] = 0.$$

If the associated positive root, $x_{i2}^0(3)$, is such that

$$x_{i2}^0(3)[1+3x_{i2}^0(3)/2] - x_2^0(2; n_1+n_2 = 1)[1+3x_2^0(n_1+n_2 = 1)/2] < \xi_2, \quad (25)$$

then i will deviate, and we will have a result analogous to proposition 1, albeit in a weaker form, whereby ξ_2 must be sufficiently large. Not all those who escaped infection in period 1 get vaccinated, but the possibility that a mixed strategy equilibrium exists is not ruled out when the inequality holds. Then again, it is possible that the setting is one wherein (25) is violated, so that i does not deviate and all susceptible individuals choose vaccination. Yet this does not also rule out the existence of an equilibrium of the kind in proposition 5.

Imperfect immunity: the effect of an increase in ξ_2 on n_2^+

Differentiating (15) and (16) totally, and rearranging, we obtain

$$d\xi_2 = (3x_2^0(3) + 1)\left[1 - \frac{x_2^0(2)(x_2^0(2) + 1)}{x_2^0(3)(x_2^0(3) + 1)}\right] dx_2^0(3),$$

so that $x_2^0(3)$, and hence also $x_2^0(2)$, is increasing in ξ_2. Writing X_2^0 as $[x_2^0(2) - x_2^0(3)]n_2^+ + x_2^0(3)$, it is seen from (14) that an increase in $x_2^0(3)$ will induce an increase in n_2^+, unless there is an accompanying increase in the difference $x_2^0(3) - x_2^0(2)$ large enough to offset the increase in all the terms involving $x_2^0(3)$ alone. Such an increase in $x_2^0(3) - x_2^0(2)$ would have to be large indeed.

Non-believers

Proceeding as in Section 2, it is readily shown that those rational individuals who decide against vaccination choose the same distancing as they would in the absence of the non-believers. For their alternative is the fixed expected cost ξ_2, and this tethers their behaviour to the indifference condition (5), which, involving only ξ_2, is independent of λ. This is the first result.

What does change is the distancing of rational individuals in period 1, together with the numbers getting infected and a jab. Since the non-believers do not distance, the fraction $p_{01} = [a(1-\lambda)x_1^* + 1]^{-1}$ of this group get infected in period 1 and so contribute $\lambda[a(1-\lambda)x_1^* + 1]^{-1}$ to the pool of naturally immune individuals in period 2. The corresponding contribution of the rational individuals is

$$n_1^r = (1-\lambda)[(x_1^* + 1)(a(1-\lambda)x_1^* + 1)]^{-1},$$

so that the the size of the whole pool of naturally immune individuals in period 2 is

$$n_1(x_1^*, \lambda) = \frac{1 + \lambda x_1^*}{(x_1^* + 1)[(a(1-\lambda)x_1^* + 1]}.$$

To show why a change in λ has an ambiguous effect on n_2^e, note that condition (10) is changed only by the introduction of the term $(1-\lambda)$ as a multiplicand of a. Recall that, after vaccination decisions, the pool of immune individuals has size $n_2^+ = n_1 + n_2^e < \underline{n}$. Differentiating totally, holding ξ_1 and ξ_2 constant, yields

$$\frac{dn_2^+}{d\lambda} = \frac{a\underline{n}x_2^*\phi_2 \cdot (1 - n_2^+)}{\xi_1 - (1-\lambda)a\underline{n}x_2^*\phi_2},$$

where $\phi_2 \equiv x_{i2}^*(x_{i2}^*+1)^2$. Hence, n_2^+ is increasing or decreasing in λ according as ξ_1 exceeds or falls short of $(1-\lambda)a\underline{n}x_2^*\phi_2$.[22] Since $x_2^*(\xi_2)$ and $\phi_2(\xi_2)$ depend only on ξ_2, the presence of non-believers makes it more likely that the size of the said pool is increasing in λ, albeit at a diminishing rate.

Now, $n_1(\xi_1, \lambda)$ is increasing in λ. Hence, if n_2^+ is decreasing therein, then n_2^e must be likewise. If, on the contrary, n_2^+ is increasing in λ, then the behaviour of n_2^e is ambiguous. Noting that $n_1(\xi_1, \lambda)$ depends on x_1^*, we have

$$\frac{dn_2^+}{d\lambda} = \frac{dn_1}{dx_1^*} \cdot \frac{dx_1^*}{d\lambda} + \frac{dn_2^e}{d\lambda}.$$

Differentiating (21), we obtain

$$\frac{dn_2^e}{d\lambda} = \frac{a\underline{n}x_2^*\phi_2 \cdot (1 - n_2^+)}{\xi_1 - (1-\lambda)a\underline{n}x_2^*\phi_2} - \frac{(x_1^*+1)(x_1^*)^2 + [(2x_1^*+1) + x_1^*(2+3x_1^*)\lambda]}{\xi_1 - \beta\xi_2} \cdot \frac{dx_1^*}{d\lambda},$$

where $dx_1^*/d\lambda$ has been derived above, as a rational function of x_1^* and λ. This demonstrates the said ambiguity: a larger pool of naturally immune individuals in period 2 may, or may not, make vaccination less attractive to rational susceptible individuals.

Appendix 2

Imperfect immunity: quantitative analysis

The endogenous variables x_1^*, $x_2^*(1)$, $x_2^*(2), x_2^*(3)$, n_1 and n_2^e are related by

$$x_1^*(x_1^*+1)(ax_1^*+1) = p(0)\xi_1(0) - \beta\xi_2, \tag{26}$$

[22] When they are equal, the derivative is undefined.

$$x_2^*(1)(x_2^*(1)+1)^2[a\{n_1x_2(1)+n_2^e x_2(2)+(1-n_1-n_2^e)x_2(3)]\}+1] = p(1)\xi_1(1), \tag{27}$$

$$x_2^*(2)(x_2^*(2)+1)^2[a\{n_1x_2(1)+n_2^e x_2(2)+(1-n_1-n_2^e)x_2(3)]\}+1] = p(2)\xi_1(2), \tag{28}$$

$$x_2^*(3)(x_2^*(3)+1)^2[a\{n_1x_2(1)+n_2^e x_2(2)+(1-n_1-n_2^e)x_2(3)]\}+1] = p(3)\xi_1(3), \tag{29}$$

$$x_2^*(2)[1+3x_2^*(2)/2]+\xi_2 = x_2^*(3)[1+3x_2^*(3)/2], \tag{30}$$

$$n_1 = p(0)\left[(x_1^*+1)(ax_1^*+1)\right]^{-1}. \tag{31}$$

As in section 2.4, we perform a sensitivity analysis with respect to changes in the expected cost of an infection, which depends on immune status, and the expected cost of vaccination. We compute a baseline equilibrium for the following constellation, which conforms to that in Section 2.3:

$\xi_1(0) = 3, \xi_2 = 0.75, \xi_1(1) = \xi_1(2) = 0.3, \xi_1(3) = 3,\ p(0) = 0.8, p(1) = p(2) = 0.4, p(3) = 0.5$ and $a = 2$. The allocation is:

$x_1^* = 0.45028, n_1^* = 0.29024, n_2^e = 0.38298, x_2^0(2) = 0.07361, x_2^0(3) = 0.48251.$ Note that full vaccination with $n_2^e = 1 - n_1^* = 1 - 0.29024$ is also an equilibrium, whereby all individuals choose the yet smaller $x_2^0 = 0.0581$.

The rationale for choosing these parameter values is as follows. At date 1, when all individuals lack any immunity, the expected cost of an infection, $\xi_1(0)$, is high. If infected in period 1 or vaccinated at date 2, the said cost in period 2 is considerably lower. The probability parameters $p(k)$ also preserve that ordering. We assume that the expected cost of vaccination is at most one eighth of the expected cost of an infection.

The results for an increase in $\xi_1(0)$ of up to 5% from its baseline level are as follows.. As in the case of herd immunity, individuals distance more in period 1, thus lowering n_1. Unlike the herd immunity case, however, distancing in period 2 depends on $\xi_1(0)$; for unlike (5), the indifference condition (15) is not anchored, there being now two endogenous variables in play, although the changes are quantitatively negligible for the 5% variation in $\xi_1(0)$. The fall in n_1 is accompanied by a rise in n_2^e, and the probability that susceptible individuals will get a jab also rises.

The results for an increase in the expected cost of vaccination are as follows. As in the herd immunity case, individuals distance less in period 1 and more in period 2, wherein the response of group S_3 differs sharply from that of groups S_1 and S_2. These responses are jointly determined with n_2^e. Since the response of $x_2^*(3)$ is much stronger than than that of $x_2^*(1) = x_2^*(2)$, this may raise the cost of social distancing for unvaccinated individuals so much – due to the strict convexity of the cost function – that more susceptible individuals opt for vaccination, thus preserving indifference in equilibrium.